DESIGN OF A THREE-POINT HITCH

SVANTE JOHANSSON

DEPARTMENT OF MECHANICAL ENGINEERING

MASTER ´S THESIS PROJECT
Department of Management and Engineering
LIU-IEI-TEK-A--08/00433--SE
Abstract

The original objective of this project was to design a three-point hitch for Buhler Versatile’s high horsepower tractors series phase C and phase D. Main guidelines for the design were to focus on simplicity and cost effective solutions. The outline of the project eventually narrowed down to focusing on a heavy duty version strictly for tractor series phase D.

The hitch is to meet with current agricultural standards and match market leading heavy duty three-point hitches in lift capacity. The targeted a lift capacity is bumped up from 13,000lb to 20,000lb and a pull capacity set to correspond with tractors working in the region of 500HP.

The report reviews the current design for possible improvements and from there, starts re-designing a new design from scratch. A computer model is created for force and critical stress analysis on the linkage system using iterative modifications of the input data. These calculations for acting forces along with stress analysis of classic machine design failures acts as a foundation of the design process. The design is also analysed with FEA, using Autodesk Inventor built in software. A design is proposed ready for prototyping where the prototype is to endure extensive testing for further evaluation outside the scope of this project.

For the design, the attachment points of the linkage to the rear frame are joined into one big weldment as per Buhler Versatile preference. For lateral constrain it is concluded to use stabilizers instead of the current sway blocks. The stabilizer brackets are linked with the main assembly using pin joints offering the lower link attachment extra support and alignment of the two links. The upper rock arm is also proposed as matching the original objective to go away from the current spline system by using a tube weldment.

Overall, the presented design meets with the targeted simplicity and cost effectiveness that characterise the Buhler Versatile brand. Remaining now is to execute extensive field testing of a prototype for further improvements to target an optimized and reliable product.
# TABLE OF CONTENTS

1  Introduction ........................................................................................................................... 5
   1.1  History ............................................................................................................................... 5
   1.2  Problem Background ........................................................................................................ 5
   1.3  Project Objective ............................................................................................................... 6
   1.4  Target Specifications ........................................................................................................ 6
   1.5  Deliverables ...................................................................................................................... 6

2  Basic Concept for design of a Three-Point Hitch ................................................................. 7
   2.1  Terminology ..................................................................................................................... 7
   2.2  Testing for Lift Capacity .................................................................................................. 8
   2.3  Buhler Versatile Testing ................................................................................................ 8

3  Analysis of Current Design ................................................................................................... 9
   3.1  Current Design ................................................................................................................ 9
   3.2  Current Design Explode View .......................................................................................... 10
   3.3  Analysis of Measurements compared to ASAE Standards .............................................. 14
   3.4  Experienced Failures ....................................................................................................... 15
      3.4.1  Lift Rod Failures ....................................................................................................... 15
      3.4.2  Upper Link Failure .................................................................................................. 16
      3.4.3  Outer Stop and Sway Block .................................................................................... 16

4  Implementation Analyzes ....................................................................................................... 17

5  Initial Work Breakdown .......................................................................................................... 18

6  Computer Model .................................................................................................................... 19
   6.1  Geometry .......................................................................................................................... 20
   6.2  Forces ............................................................................................................................... 21
   6.3  Stresses ............................................................................................................................. 21

7  Positioning and Geometry ..................................................................................................... 22
   7.1  Basic Geometry ................................................................................................................ 22
   7.2  ASAE Standard Measurements ...................................................................................... 23
   7.3  Positioning of Pivoting Points ......................................................................................... 24
   7.4  Conclusions of Constraints in Summary ....................................................................... 24

8  Design of Key Components .................................................................................................. 25
   8.1  Pins at Pivot Points ......................................................................................................... 25
   8.2  Upper Arm ...................................................................................................................... 27
   8.3  Hydraulic Cylinder ......................................................................................................... 32
1 Introduction
This is a master’s thesis project report for the department of mechanical engineering at Linköping University, Sweden. The project objective was to design a three-point hitch for the Buhler Versatile 4WD tractors Phase C2 and Phase D. The project was fully conducted at Buhler Versatile facilities where Design Engineer Matt Rhyner and Head of Engineering Allan Minaker supervised the design process.

1.1 History
Buhler Industries was established in 1979. It designs and manufactures agricultural machinery for use in small and large-scale farming. Products include tractors, wheel loaders, augers, cultivators, landscape rakes and mowers. Buhler also makes snow blowers and belt conveyors. The tractor division Buhler Versatile started in the year 2000 when it took over the facilities, and products, from New Holland Versatile in Winnipeg.

1.2 Problem Background
To meet with today’s market demand for agricultural equipment of larger implements; Buhler Versatile has over the past couple of years rapidly increased engine power on their 4WD tractors. The current three-point hitch system, for category 3 and category 4N, has however not been updated to match today’s tractor capacity. The current three-point hitch was originally designed for a 300HP engine, but is still being implemented with newer engines working in the region of 500HP. The design now needs to be strengthened to remain durable. The target is to increase the lift capacity from 13,000lb to 20,000lb.
In addition to strengthening the design, the feasibility of altering the design for the upper arm is preferred. The current design is manufactured by three cast parts which is connected in a spline system. The feasibility of not using cast parts and altering spline system to a linking weldment are to be considered. Basically the entire design of the three-point hitch is to be looked over and designed from scratch. The final product is to meet with ASAE Standards for agricultural equipment.
1.3 Project Objective

Produce a full design, all set for manufacturing of a prototype.

1.4 Target Specifications

1.4.1 The design shall have a lift capacity of 20,000Lb in accordance with ASAE\(^1\) standard “SAE J283 NOV” (Test Procedure for Measuring Hydraulic lift Capacity on Agricultural Tractors Equipped with Three-Point Hitch).

1.4.2 The geometry, basic dimensions and the moving pattern are to meet with the ASAE Standard “ASAE S217.12 DEC01” (Three-Point Free-Link Attachment for Hitching Implements to Agricultural Wheel Tractors). The design is to meet with specifications for category 3, category 4N (narrow) and category 4.\(^2\) It is to meet clearance zones described in ISO standard, “ISO 2332-Agricultural tractors and machinery – Connection of implements via three-point linkage”. It is also to meet with specifications for U-frame coupler in accordance with ASAE standard, ASAE S278.7 JUL03 (ISO 11001-1 1993).

1.4.3 The safety factor of 3 is to be applied to the design.

1.4.4 The feasibility of not using a spline system nor cast parts for the upper arm is to be considered. The feasibility of linking the arms with a tube weldment is to be considered.

1.4.5 A hydraulic working pressure of approximately 2750psi is to be targeted for a max loading scenario.

1.4.6 Manufacturing methods should preferably be considered within Buhler’s range of manufacturing capability.

1.4.7 Material Mild Steel ASTM A36 is preferred.

1.5 Deliverables

Engineering Drawings

---

\(^1\) American Society of Agricultural Engineering

\(^2\) Categories revised, please refer to section 4
2 Basic Concept for design of a Three-Point Hitch

2.1 Terminology
Presented below is a list of basic names for the key components that is used throughout this report. The hitch system is divided into following components in accordance with standard “ASAE S217.12 DEC01”.

**Key components and measurements on a three-point hitch**

1. Upper Link
2. Lower Links
3. Upper Link Hitch Point, ULHP
4. Lower Link Hitch Points, LLHP
5. Upper Link Point, ULP
6. Lower Link Points, LLP
7. Upper Hitch Attachment
8. Lower Hitch Attachment
9. Upper Link Attachment
10. Linch Pin
11. Lift Rods
12. Mast
13. Mast Height
14. Mast Height
15. Leveling adjustment
16. Lower Hitch Point Span
17. Linch Pin Distance
18. Movement Range
19. Transport Height
20. Lower Hitch Point Clearance

Number 15-20 are measurements to meet with ASAE Standards.
2.2 Testing for Lift Capacity

Figure 2.3: Loading scenario for measuring lift capacity

A lift capacity of 20,000Lb is to be targeted. The final product is to meet with this load being tested both for static and for dynamic loading scenarios in accordance with ASAE standard “SAE J283 NOV99”. The lift capacity is determined from the load resulting in maximum specified hydraulic pressure.

2.3 Buhler Versatile Testing

Buhler Versatile Prototype testing include a fully loaded “bump test” where the load increases up to 3G. This testing is considered to give assurance of dynamic loading. Further testing involves a cycle-test where the maximum hydraulic pressure is applied to the linkage with the hitch points fixed. The maximum pressure is applied for 5 seconds followed by 5 seconds of rest in a cycle process. 50,000 cycles is targeted minimum before fatigue failure. Note that this is not a lift cycle since the linkage is fixed at the hitch points.

The prototype is also to be exposed to a full season of field work before put into production.
3 Analysis of Current Design

3.1 Current Design

Figure 3.1 shows the current three-point design attached to a tractor series Phase C2. Figure 3.2 illustrates that tractor Phase D has a wider rear frame. The only part that differs is however the attachment of the lower links and the size of the drawbar cage (to which the drawbar and the hydraulic cylinders are mounted). The design is interchangeable for category 3W (wide) or category 4N (narrow). Both frames are equipped with similar hole-pattern.

![Figure 3.1: The current design on a Phase C2](image1)

![Figure 3.2: Rear frame Phase D](image2)
3.2 Current Design Explode View

Figure 3.3 shows the current design in an explode view. The new design is to have an option that takes into account extra spacing for a PTO (Power Takeoff Output). This version is shown in the "W/PTO-box".

Figure 3.3: Explode view of the current Three-Point Hitch Design
Brief Review of Components (explode view number in parenthesis)

**Drawbar Cage (1):** The drawbar cage mounts the crossbar which gives extra support to the drawbar in accordance with drawbar standard “ASAE S482 FEB04”. It also holds the two hydraulic cylinders (3). The crossbar positioning and its thickness differ between the two tractors Phase C2 and Phase D. The height from the bottom of the crossbar above the ground is set to 22 inches for both tractors in accordance with ASAE Standard [2]. The drawbar has suffered failure at weld when welded to the inner side of the drawbar cage side plate. This has been altered to have the drawbar go through the plate and welded on the outside of the cage for extra strength. For assembling, a socket will not fit the outer bolts due to its located too close to the plate (Consultation with Norm Huley, Head of Assembling at Buhler Versatile).

**Rock Shaft (2):** The Rock Shaft connects the two upper rocking arms (4) with a spline system. This is preferably to be altered to a connecting tube weldment. The feasibility of this by pivoting the Upper Arms around two pins is to be considered. A sensor measuring the Rock Arm positioning for hydraulic control is positioned at the very end of the Rock Shaft assembly.

**Hydraulic Cylinder (3):** The tractor can produce up to 3000psi of pressure. To assure not exceeding this pressure, the hydraulic cylinders are equipped with a relief valve set to 2895psi [3]. The cylinder is double acting but is implemented as single acting with the current control system. For lowering, it is relying on its own down force to retract the cylinders. This has caused problems due to the return system pressure being too high. If an implement is not attached, it is required to apply full body weight to lower the links for attachment. A double acting cylinder is therefore preferred to have an option of future modifications to the hydraulic control (the design of hydraulic control falls outside the scope of this project). The cylinders are mainly loaded in compression meaning the piston rod side surface area can be neglected for extreme loads.

**Upper Arm (Rocking Arm) (4):** The structure of the Upper Arms is preferably altered to a more cost effective solution than the present cast pieces and spline system. The current version does not align the Hydraulic Cylinder (3) and the Lift Link (14) creating a torque upon the arm and spline. A design with the two components aligned is to be compared with the current version. The feasibility of pivoting the arms around two pins is to be analyzed. A sensor measures the angle of the Upper Arm for the hydraulic control.

**Bracket Assembly (13):** The Bracket Assembly holds the attachment point for the Lower Link (15) to the rear frame. This design does not fall within the specifications of ASAE standard for Lower Link Convergence. This version does not have any convergence even though ASAE standard specifies that the convergence is of great importance to the implements stability [4]. It also holds the outer stops for constraining the lower links movement in a horizontal plane to the ground. There has been problems experienced with the outer stops being too weak and it is suggested to be strengthened. The outer stops are however for the lower link to stay clear of the tires when there is no implement attached at the hitch points. When an implement is attached, the Sway Blocks (35) acts as the outer stop. It is suspected that failure of the outer stop is caused due to attachment of an implement wider than the specified dimensions for the hitch. A wildly used
solution for this is a spring tying the lower links together causing the sway blacks to interfere prior to the wheels. The use of stabilizers instead of sway blocks (35) would eliminate this problem.

**Lift Rod (14):** The Lift Rod is the weakest link on the current design. It needs to be significantly strengthened to meet with the new lift capacity. The structure of the Lift Rod consists of two threaded rods that connect in a connecting adjustment tube. The length of the Lift Rod can be adjusted by turning the adjustment tube where the threaded rods (of opposite thread LH and RH) extends or retracts. For the current version, the Lift Rod connects to the Lower Link by a pin through a hole or a slotted hole. The slot is used to give the implement torsional free float in case of, for example hitting rocks or having an implement greatly wider than the tractor wheels. On the current version the slot and the hole does not align with the rod center line creating a bending moment on the Lift Rod. This fatigue failure is considered to be the weakest point of the current design. The current versions solution for adjusting the length of the Lift Rod is poor. The leaver is ungainly for adjusting the length, and there is no option for using another tool, for example a hex- or a square-geometry for a wrench. Also, the leaver attaches by a pin going through the adjustment tube (part connecting the two threaded rods). Stress concentration at this hole makes it a week point for loading. The hole is however also used for locking the threaded parts from detaching the assembly.

**Lower Link (15):** The current Lower Link attachment point to the rear frame needs to be adjusted for greater convergence. Also, the moving pattern for the Hitch Points must meet with standardized measurements in accordance with ASAE Standards. The current Lower Link has two settings for attaching depending on the implement category. The hitch point ball width for category 3 differs from category 4 and therefore, the Lower Link is equipped with two different ball sizes at each end. The categories differ in both inner diameter and width. The Lower Link is to be flipped around depending on implement cat 3 or implement cat 4N. When cat 3 is used, due to the smaller pin diameter at the hitch points [4], a bushing must be used. This is a good solution to get around the problem, but rarely serves its purpose since the execution is very laborious.

**Upper Link Attachment (24):** The Upper Link Attachment varies for with, or without PTO. There are two settings for the Upper Link, the lower setting is only for category 3 and the upper hole is for both category 3 and 4N. This is not specified in ASAE Standards for categories, but two settings are specified for pitch adjustment of the mast [4].

**Upper Link (27):** The Upper Link attaches to the top of the implement mast and by adjusting the link’s length, the implements mast angle varies. The Upper Link must both meet with standardized measurements for mast pitch and have sufficient vertical convergence for the implement stability. When the hitch is not in use the Link is fastened to the Upper Link Attachment. This is currently poorly designed where it rattles and hits the hydraulic output couplers.

**Sway Blocks (35):** The sway blocks function is to constrain the lateral movement of the implement in a horizontal plane to the ground. There are two settings for the sway block. Either sway mode where the Lower Links are given room to sway in free float at a lowered position and gets centered for minimum sway at a raised positioning (transport height). The other option is minimum sway
mode. For this, the sway blocks are flipped around giving minimum sway for all positioning. This setting is a must if the PTO is in use.
The sway blocks are also used to clear the lower links from the tires when an implement is attached. There have been complaints of the sway blocks rubbing too much. Therefore, different materials for a more durable product should be considered.
Also, sway blocks are mostly used for the North American market. For the European market, the use of “stabilizers” is more common. The stabilizer is a link that attaches to the lower link and restricts it in a similar manor as the sway block to achieve the standardized hitch point movement. Stabilizers are considered a better solution where sway blocks are more cost effective.
3.3 Analysis of Measurements compared to ASAE Standards

The current version is analyzed for compatibility with the current ASAE Standards.

ASAE Standard specifies the following key measurements:
For detailed information please refer to ASAE S217.12 DEC01.

Table 3.1: Standardized measurements on the current design

<table>
<thead>
<tr>
<th>Measurement:</th>
<th>Current Design [mm]</th>
<th>ASAE Standard [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Hitch Point Height</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>Movement Range</td>
<td>740</td>
<td>760</td>
</tr>
<tr>
<td>Leveling Adjustment</td>
<td>205</td>
<td>150</td>
</tr>
<tr>
<td>Transport Height</td>
<td>1380</td>
<td>1200</td>
</tr>
<tr>
<td>Hitch Point Clearance</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>Horizontal Convergence</td>
<td>-</td>
<td>1900-2800</td>
</tr>
<tr>
<td>Vertical Convergence</td>
<td>1800</td>
<td>3540</td>
</tr>
<tr>
<td>Hitch Point to PTO</td>
<td>717</td>
<td>675-775</td>
</tr>
<tr>
<td>Torsional Free Float</td>
<td>70</td>
<td>75</td>
</tr>
<tr>
<td>Mast Adjustment Min</td>
<td>450</td>
<td>255</td>
</tr>
<tr>
<td>Mast Adjustment Max</td>
<td>805</td>
<td>710</td>
</tr>
</tbody>
</table>

Comments

**Movement Range**
The current version does not meet with the ASAE Standard of minimum movement range.

**Horizontal Convergence**
The current version does not have any convergence. The ASAE Standards specifies a range of 1900-2800mm for sufficient stability.

**Vertical Convergence**
The current version has a vertical convergence of 1800mm. This does not meet with the ASAE recommendations of minimum 3540mm (0.9 times wheel base).

**Hitch Point to PTO**
The current version has a distance of 717mm between the hitch points and the PTO when the lower links are level to the ground in a horizontal plane. The ASAE standard specifies a rage of 675-775mm for a PTO diameter of 45mm. This was recently altered to meet with ISO standards for the European market where the previous standards were 575-675mm. ASAE recommends designing around the minimum distance in the new range to meet with both new implements and the old North American implements.

**Torsional Free Float**
The current version has a torsional free float that does not meet the ASAE Standard of minimum torsional free float.

**Mast Adjustment**
The current version has a lower hitch point height of 450mm and a top of 805mm for the 5° mast adjustment. The lower height is not adequate to meet with ASAE standards.
3.4 Experienced Failures

3.4.1 Lift Rod Failures

Figure 3.4-3.7 shows documentations of different failures.

**Figure 3.4** shows failure at the bolt attachment of the leaver. The stress concentration at the hole for the bolt greatly reduces the strength of the link from a load perspective. The leaver attachment can be solved without using a bolt through the connector but the hole is also used for prohibiting the threaded rods from detaching the rod attachment. The material thickness of the adjustment tube needs to be significantly scaled up.

**Figure 3.5** illustrates a failure of the pin that attaches the lift rod to the rock arm. This pin joint is not lubricated where the friction forces action on the pin results in a torque on the pin. Simple grease lubrication is estimated to give sufficient friction reduction to solve the problem. For the coating of the pin, zinc plating is used. Zinc plating tends to wear off after long wear causing more friction on the pivot. An improved coating such as chrome or salt bath nitride is however considerably less cost-effective. Preferably reduction of contact friction in the pin joint is preferred.

**Figure 3.6** shows a homemade reinforcement of the Lift Rod attachment to the Upper Arm. The transition from threaded rod to the ball head is too weak and it needs to be significantly strengthened. The upper link has already been supported with gussets welded on, similar to figure 3.10 and the lift rod should not be an exception.

**Figure 3.7** illustrates failure at the threaded rod, more than likely caused due to the bending moment of not aligning the upper and lower attachment points.
3.4.2 Upper Link Failure

The upper link has experienced problems at the transition from threaded rod to the attachment points. Figure 3.8 and 3.9 shows homemade reinforcements of the attachment to the implement and the rear frame. The current version is however updated. It is currently reinforced with gussets at the transition between threaded rod and ball head, similar to the figures.

3.4.3 Outer Stop and Sway Block

The Outer Stop that prevents the Lower Link from hitting the tire is too weak and is suggested to be strengthened. Figure 3.11 illustrates a homemade version with welded reinforcements. The outer stops are however only supposed to be used when there is no implement attached to the linkage (no weight). When an implement is attached, the Sway Block acts as the Outer Stops to clear the tire from interference. It is suspected that the failure of the outer stops could be the result of using an implement with a hitch point span wider than the specified range. This could cause the lower links to hit the outer stops before the sway block interfere. Possible alternate solutions could be to link the lower links with a spring constraining the hitch point span or the use of stabilizer links. Also, there have been complaints about the sway blocks rubbing down too fast. Harder material than today’s grey cast iron should be considered.
4 Implementation Analyzes

It is initially desired to have a similar design for both tractor Phase C2 and tractor Phase D. Even though the two tractors differ in width of the rear frame, they both basically have the same hole-pattern for attachments. The new design is preferred to be interchangeable between Category 3, 4N and 4. Investigation of implementing such a design however confirms that this is not possible without compromising the ASAE Standards. For example the ASEA standard specifies different ball sizes at the hitch points for cat 3 and cat 4. Also, even if the hitch point span differs widely between cat 3 and 4, they still both have the same specified convergence distance, 1900-2800mm [4].

For the convergence, both category 3 and 4N is similar and can easily be designed to meet within the specified range. There is however a design issue where the standards does not specify the same ball width at the hitch points for both cat 3 and cat 4. The current design has a system where the Lower Link has a cat 3 ball at one end and a cat 4 at the other. This gets around the problem but the conversion is however a laborious procedure. A U-frame coupler conversion is considered better since it gives field-interchangeability. U-frame coupler standardized specifications give design of interchangeability between cat 3 and cat 4N [6].

Further analyzing the categories, when designing for both cat 4N and cat 4, the ball size stays the same at the hitch points but unfortunately the convergence has to be compromised. For example: If category 4 lower link point span is designed for the maximum convergence, 1900mm, a category 4N implement will have a convergence of 5500mm (Far out of the range). It is possible to have a design were both categories fall out of the range, for example: when cat 4 has a convergence of 1600mm, cat 4N has a convergence of 3700mm. (Values are calculated for a Lower Link length of 975mm).

The ASAE mentions that the convergence is of great importance for the stability of the implement. The current version however does not have any convergence and it is hard to estimate the reduction in quality by compromising the convergence.

Further implementation analysis, Category 4 specifies a minimum horizontal hitch point movement distance of 610mm plus/minus 130mm from the tractor centerline. This will interfere with the tractor tires on the Phase C2 unless the convergence is compromised. Phase D is however wider and has sufficient clearance.

As per discussion with Head of Engineer Al Minaker the following conclusions are made: a design strictly for category 4 is to be designed for tractor series Phase D. The design is to be interchangeable with category 4N, but mainly optimized for standard category 4 implements. This option is only to be offered for tractor Phase D and is planned to be interchangeable with Category 4N and Category 3 using a Quick Coupler. This project main target is set to design a category 4 three point hitch system for the tractor series Phase D.

| Table 4.1: Specified Standards |
| Hitch Point Spans [mm] |
| Cat 3   | 965 |
| Cat 4N  | 920 |
| Cat 4   | 1165 |
5 Initial Work Breakdown
The task is divided into subsections to break down the design process. The subsections are described in sequential sections.

6 Computer model: A computer model is constructed for iterative analysis of the design. The model is used for quickly re-analyzing the design with modified input data.

7 Positioning and Geometry: Initial positioning of the attachment points and the measurements of the components in the linkage system is set. This is optimized by reducing the acting forces within the constraints of standardized measurements. Concluded values are only to be considered as an initial reference where iterative adjustments of the positioning are to be made throughout the design process.

8 Design of Key Components: Upper Arm, Lift Rod, Lower Link, Hydraulic Cylinder and Upper Link. Each component is considered with numerous different designs to be analyzed for implementation. The components and their pivoting pins are dimensioned using both FEA combined with basic stress analyzes for critical points. The different solutions are rated for cost effectiveness, manufacturing aspects, assembling aspects, weight and appearance.

9 Design of Attachments: The attachments for the Upper Arm, the Upper Link, the Lower Link, the Hydraulic Cylinders and the Draw Bar. (The Lower Link, the Hydraulic Cylinders and the Draw Bar are all attached in the same assembly, the “Draw Bar Cage”). Stress analysis is conducted using Autodesk Inventors basic FEA and basic stress calculations. The components are analyzed and rated for cost effectiveness, manufacturing aspects, assembling aspects, weight and appearance.

10 Proposed Design: The proposed design in summary.
6 Computer Model

A mathematical model is setup for the key components in the linkage system. From specified input data, the model calculates the moving pattern of the components. By knowing the moving pattern, the acting forces and stresses can be calculated for each specific positioning. The model is used to quickly examine different structures and geometry for the pivoting points. The linkage system is optimized by both analyzing resulting formulas and by iterative alterations of the input data to improve the results. The input variables are optimized to reduce the acting stresses on the components for targeted lift capacity. This while not compromising the standardized constraints of the moving pattern.

The calculations only take into account loadings in a simplified 2D scenario. The resulting stresses are to be viewed as rough estimations and are not to be considered exact scientific values.

For detailed calculations please refer to Appendix A.
6.1 Geometry

The components are analyzed as vectors in an approximated 2D scenario. The Lift Rod and the Lower Link are modified to resulting 3D-values when considered necessary. The Computer model has coordinates for the pivoting points and the components lengths as input data and the components positioning as output data. From set pivoting points, adjustment in length of the Hydraulic Cylinder, Lift Rod and the Upper link gives the moving pattern in an approximated 2D plane.

For detailed information please refer to Appendix A

![Figure 6.1: Components as vectors](image)

Table 6.1: Input and Output variables in the linkage system.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Input/Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Coordinates of Lower Link Point</td>
<td>In</td>
</tr>
<tr>
<td>UL&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Coordinates of Upper Link Point</td>
<td>In</td>
</tr>
<tr>
<td>UA&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Coordinates of Upper Arm Point</td>
<td>In</td>
</tr>
<tr>
<td>HC&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Coordinates of Hydraulic Cylinder Point</td>
<td>In</td>
</tr>
<tr>
<td>LL&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from LL&lt;sub&gt;p&lt;/sub&gt; to LL&lt;sub&gt;HP&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>UL&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from UL&lt;sub&gt;p&lt;/sub&gt; to UL&lt;sub&gt;HP&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>LR&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from LLLR&lt;sub&gt;p&lt;/sub&gt; to UALR&lt;sub&gt;p&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>M&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from LL&lt;sub&gt;HP&lt;/sub&gt; to UL&lt;sub&gt;HP&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>HC&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from HC&lt;sub&gt;p&lt;/sub&gt; to UAHC&lt;sub&gt;HP&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>UAH&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from UA&lt;sub&gt;p&lt;/sub&gt; to UAHC&lt;sub&gt;p&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>UAL&lt;sub&gt;length&lt;/sub&gt;</td>
<td>Distance from UA&lt;sub&gt;p&lt;/sub&gt; to UALR&lt;sub&gt;p&lt;/sub&gt;</td>
<td>In</td>
</tr>
<tr>
<td>LL&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Coordinates of Lower Link Hitch Point</td>
<td>Out</td>
</tr>
<tr>
<td>UH&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Coordinates of Upper Link Hitch Point</td>
<td>Out</td>
</tr>
<tr>
<td>LLLR&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Coordinates of the Lift Rod attachment to the Lower Link</td>
<td>Out</td>
</tr>
<tr>
<td>UALR&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Coordinates of the Lift Rod attachment to the Upper Arm</td>
<td>Out</td>
</tr>
<tr>
<td>UAHC&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Coordinates of the Hydraulic Cylinder attachment to the Upper Arm</td>
<td>Out</td>
</tr>
<tr>
<td>LL&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking LL&lt;sub&gt;p&lt;/sub&gt; to LL&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>UL&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking UL&lt;sub&gt;p&lt;/sub&gt; to UL&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>HC&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking HC&lt;sub&gt;p&lt;/sub&gt; to UAHC&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>M&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking LL&lt;sub&gt;H&lt;/sub&gt; to UL&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>LLLR&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking LL&lt;sub&gt;p&lt;/sub&gt; to LR&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>UAHC&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking UA&lt;sub&gt;p&lt;/sub&gt; to UAHC&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
<tr>
<td>UALR&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Vector linking UA&lt;sub&gt;p&lt;/sub&gt; to UALR&lt;sub&gt;H&lt;/sub&gt;</td>
<td>Out</td>
</tr>
</tbody>
</table>
6.2 Forces

The acting forces on the components are calculated from the loading scenario described in section 2.2 “Testing for Lift Capacity”. For the upper arm, the loading scenario is divided up into two alternative loading scenarios:

**Alternative #1:** The Lift Link and Hydraulic Cylinder do not align with a resulting torque.

**Alternative #2:** The Lift Link and the Hydraulic Cylinder align in a plane with a resulting lateral force.

Also, an estimation of an extreme pulling scenario is evaluated.

*For detailed information regarding calculations and free body diagrams please refer to Appendix A*

6.3 Stresses

All pivoting pins are analyzed and the component stresses are analyzed for critical points. The calculations analyze a simplified scenario and should be considered as a rough estimation. The designing is based on resulting values in correspondence with the FEA to fall within the material properties. The main procedure to assure design safety is however prototype testing. Different structures of the upper arm are compared and evaluated for the final design.

*Please refer to Appendix A for detailed information regarding the calculations*
7 Positioning and Geometry

7.1 Basic Geometry

The current design of the three point hitch has an Upper Arm system where two rocking arms are synchronized by a spline system. This is preferably altered to a system where the two arms pivot around pins and are synchronized with a linking weldment. The design suggested is to have two flame cut sheet metal plates acting as rock arms where a tube weldment synchronizes the arms. This simple option is to be compared with an alternative of a solid cast piece. Also, the pivot points of the hydraulic cylinder and the Lift Rod does not align on the current version creating a torque upon the arm. The advantages by aligning the three components are analyzed and compared with the current design.

The Lower Links on the current design does not meet with the ASAE Standard for convergence. This convergence has proven to be of great importance to the implement stability and cannot be neglected [4]. Due to the gain of greater convergence the lower link point span must be reduced. This intervenes with the positioning of the hydraulic cylinder and therefore, the positioning of the hydraulic cylinder requires moving. For the alignment, the hydraulic cylinder is raised to fit in between the upper arm and the lower link in an aligning plane. This is considered the best solution and is the most common solution for hydraulic lifted three point hitches in general.
7.2 ASAE Standard Measurements

The design is to meet with category 4 stated in ASAE Standard “ASAE S217.12 DEC01”. The Standard was revised DEC 2001 to correspond with “ISO 730-1:1994”, meaning that a design that meets with the ASAE Standard also meet with ISO standard [7].

The standard for category 4 is divided up into two sub categories, cat 4L (light duty) and cat 4H (heavy duty). The two categories are very similar but differ in: required movement range, mast height and range of the PTO distance. The design is concluded to mainly be designed for cat 4L but to be interchangeable with cat 4H. For interchangeability between the two categories, the main concern is that cat 4H attaches implements with a mast height of 1100mm. The option of an extra setting for the upper link position is to be compared with the option of a different Upper Link attachment that can be offered for greater mast heights.

For other divergence concerning the two categories, cat 4H increases the minimum movement range from 760mm to 900mm. Increasing the movement range however not only makes the design interchangeable with cat 4H implements but also results in the convenience of less manual adjustment required for varying implements. The minimum range for PTO distance is reduced for cat 4H but only slightly.

Specified standards by ASAE concerning the positioning and the geometry

<table>
<thead>
<tr>
<th>Specified Standard</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTO Distance</td>
<td>The ASAE specifies the distance for the lower link hitch point to the PTO when the Lower Links are horizontal to the ground. The range is recently converted to match ISO standard and should be designed for the minimum value to be compatible with older implements for the North American market.</td>
</tr>
<tr>
<td>Lower Hitch Point Height</td>
<td>The maximum height of the lower hitch-point above the ground when the hydraulic cylinder is fully lowered in conjunction with the lift rod being fully extended.</td>
</tr>
<tr>
<td>Transport Height</td>
<td>The maximum height of the hitch points when the hydraulic cylinders fully raised and the lift rod fully retracted.</td>
</tr>
<tr>
<td>Movement Range</td>
<td>The minimal vertical movement of the hitch points with no adjustment of the Lift Rods.</td>
</tr>
<tr>
<td>Mast Adjustment Height</td>
<td>In between a specified range above the ground the mast pitch is required to be able to adjust in the range of ± 5°.</td>
</tr>
<tr>
<td>Horizontal Convergence</td>
<td>The horizontal convergence is set from the adjustment of the Lower Link Point span.</td>
</tr>
<tr>
<td>Vertical Convergence</td>
<td>The vertical convergence is set by the positioning of the Upper Link attachment in a vertical plane.</td>
</tr>
<tr>
<td>Lower Hitch Point Clearance</td>
<td>The minimum clearance to the tires.</td>
</tr>
</tbody>
</table>

3 ISO - INTERNATIONAL STANDARDS ORGANIZATION
7.3 Positioning of Pivoting Points

The positioning of each component is analyzed for constraints and from there, optimized for moving pattern and stress reduction. This is done for initial coordinates where the final positioning is set after iterative adjustment during modeling. The process is briefly documented in Appendix B and concluded in conjunction with iterative modeling.

7.4 Conclusions of Constraints in Summary

Conclusions for the geometry and initial working coordinates are briefly described in Appendix B and are only to be considered as a reference for initial modeling.

Key measurements are finally set as follows:

- The positioning of the pivoting points are set in coordinates as follows:

\[
\begin{align*}
LL_p &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
HC_p &= \begin{bmatrix} 30 \\ 209 \end{bmatrix} \\
UA_p &= \begin{bmatrix} -150 \\ 880 \end{bmatrix} \\
UL_p &= \begin{bmatrix} 238 \\ 675 \end{bmatrix} \text{ [mm]}
\end{align*}
\]

- The attachment LLP is set to be located 600mm above the ground.
- The PTO distance to LLHP is set to 730mm.
- The Lower Link length is set to 975mm.
- The Lower Link Point span is set to 850mm.
- The angle of the Lower Link in a fully raised positioning is set to 40°.
- The distance between LLP and LLLR is set to 670mm.
- The Hydraulic Cylinder retracted length is set to 600mm.
- The Hydraulic Cylinder stroke is set to 295mm.
- The distance between UAP and UALR is set to 600mm.
8  Design of Key Components

8.1  Pins at Pivot Points
For a load failure, the pivoting pins are to be designed as the weakest link of the complete design. The components are to be designed using a safety factor of 3 where the weakest point is to be designed roughly around a factor of 2. It is concluded that the Lift Rod attachment to the Lower Links is the best point for failure causing least damage to the hitch system. The Lift Rod is therefore divided up in to lower attachment, LR, and upper attachment, LRU.

All pins are to be hardened and plated with a zinc-steel alloy to take the shock of heavy loads and withstand wear from implements. This is to be conducted in accordance with Buhler Versatile standard 86512106 (BAKE PER STD 86508305) and 86508316 (ZN YCR STD 86508305).

All pivoting pins are calculated for maximum shear described in Appendix A. The pin material is selected to match Buhler standard pins, 86050098 STK-AISI C1045 CD RD. The pins are to be evaluated for yield strength of 345MPa (50,000psi).

Iterative testing of varying positioning of the linkage system compared with the pulling scenario gives the maximum stresses at the following positions for the pivots.

<table>
<thead>
<tr>
<th>Component</th>
<th>Positioning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Link LLP</td>
<td>Lowered (Pulling)</td>
</tr>
<tr>
<td>Upper Link ULP</td>
<td>Lowered (Pulling)</td>
</tr>
<tr>
<td>Lift Rod LR</td>
<td>Fully Raised</td>
</tr>
<tr>
<td>Hydraulic Cylinder HC</td>
<td>Fully Raised</td>
</tr>
<tr>
<td>Upper Arm UAP</td>
<td>Fully Raised</td>
</tr>
</tbody>
</table>

For standard sized pins, each component is calculated for their maximum stress in accordance with table 8.1 and presented in table 8.2. The stresses are a result of either the lift or the pull scenario described in appendix A. Upper Link and Lower link is analyzed for dimensions in accordance with ASAE standard specifications (cat 4) at attachment points.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>-</td>
<td>-</td>
<td>205</td>
<td>247</td>
<td>66</td>
<td>255</td>
</tr>
<tr>
<td>1.5</td>
<td>-</td>
<td>-</td>
<td>143</td>
<td>172</td>
<td>46</td>
<td>177</td>
</tr>
<tr>
<td>1.75</td>
<td>-</td>
<td>108</td>
<td>105</td>
<td>125</td>
<td>34</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>192</td>
<td>-</td>
<td>80</td>
<td>96</td>
<td>26</td>
<td>99</td>
</tr>
</tbody>
</table>
Analysis
The weakest link of the design, LR, is to have the lowest factor of safety where the complete design is to meet with a specified overall safety factor of 3. The sizes of the pin at the hitch points are standardized and these components preferably have the same pin size at both ends. For the lower link hitch point, 2inch pin diameter is specified and for the upper link, of 2inch and the upper link, 1.75inch pin diameter is specified as standardized measurements. Table 8.3 describes the maximum sheer stresses exerted on the pivoting pins. The Lift Rod, Upper Arm and Hydraulic Cylinder are set from the ASAE standardized lift scenario. The Lower Link attachment and the Upper Link attachment are however not loaded at its full potential at the calculated lift scenario described in Appendix A. This due to when an implement is being pulled, most additional stresses gets concentrated at these points. The estimated maximum pull scenario (100lb per horse power) gives a pulling force of 220kN for a 500HP engine. This gives the standardized pin sizes at LLP and ULP to be sufficient.

The cost of increase in pin diameter can be neglected resulting in the clearance at the different pivot points weighing in more. The upper arm attachments to the rear frame, UAP, are relatively free from clearance constraints and can therefore be designed for extra strength. The Hydraulic Cylinder and the Lift Rod is however a bit restricted. In particular the hydraulic cylinder should be designed in the minimum range due to the extra gain of stroke for the cylinders. Considering this, the pivoting pins are selected to match required factor of safety.

<table>
<thead>
<tr>
<th>Point</th>
<th>Pin Diameter [in]</th>
<th>Max Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLP</td>
<td>2&quot; (50.80mm)</td>
<td>191</td>
<td>1.8</td>
</tr>
<tr>
<td>ULP</td>
<td>1.75&quot; (44.45mm)</td>
<td>79</td>
<td>3.2</td>
</tr>
<tr>
<td>LR</td>
<td>1.75&quot; (44.45mm)</td>
<td>105</td>
<td>3.3</td>
</tr>
<tr>
<td>LR_{FF}</td>
<td>1.5&quot; (38.1mm)</td>
<td>143</td>
<td>2.4</td>
</tr>
<tr>
<td>HC</td>
<td>1.75&quot; (44.45mm)</td>
<td>125</td>
<td>2.8</td>
</tr>
<tr>
<td>UAP</td>
<td>2&quot; (50.80 mm)</td>
<td>99</td>
<td>3.5</td>
</tr>
</tbody>
</table>
8.2 Upper Arm

The main objective for the upper arm is to analyze the advantage of using a tube to synchronize the arms in torsion instead of today’s spline-system. For this, the numerous options of structural geometries are analyzed and narrowed down to four alternatives as shown below in figure 8.1. The four options are analyzed and compared for cost effectiveness, manufacturing aspects and appearance.

![SOLID PLATE](image1)

![I-BEAM CASTING](image2)

![TWO PLATES](image3)

![T-BEAM CASTING](image4)

*Figure 8.1: Variety of structural geometries analyzed for the Upper Arm*

**Solid Plate**

This option is kept simple where two flame cut sheet plates acts as rock arms with a tube weldment synchronizing the two arms.

**Movement Range:** The main down side of the solid plate is the constraints of using clevis attachments for the hydraulic cylinder and the lift rods. This reduces the cylinder movement range due to extra clearance for the clevis and also the lift rod clevis constrains the upper arm in a fully raised positioning. The clearance is optimized by removing material around the pivoting points making this a weak point for loading.

**Manufacturing:** The main assembly can be manufactured in house where bushings are to be supplied. An issue may be to align holes from side to side since welding will distort the two plates after release of clamps in welding fixture. Machining after welding is however not an option. The bushings are to be press fitted at Buhler Versatile facilities.

**Cost:** This is the most cost effective option. Cost of welding fixture is little compared with tooling for a casting and can be reused for small modifications after testing of a prototype.

**Appearance:** Gives a simple and cost effective appearance.
**TWO PLATES**

This option have two flame cut sheet plates which acts as a rock arm with a two plates welded in between for support.

**Movement Range:** This option benefits the movement range since there is no need for clevis attachments that constrains the maximum raise of the upper arm.

**Manufacturing:** Can be manufactured in house. There may be problems aligning the two plates after welding the supporting plates, this is a narrow tolerance since it uses the same pin. There may also be an issue to align holes from side to side since welding will distort the two plates after release of the clamps in welding fixture. This is however wider tolerance. For anti rotation of the pins, the plates need to be equipped with a tapped hole.

**Cost:** This is a cost effective solution in terms of simple flame cut sheet plates in a weldment, but adds extra cost in comparison to a solid plate. The number of plates is increased adding extra cost of cutting and machining. The extra tapped holes add cost to the upper arm design but the difference in cost comparison to machining the hole on the clevis is negligible. Furthermore, the extra supporting plates and welding adds cost.

**Appearance:** Gives a somewhat cheep and temporary appearance.

**I-BEAM CASTING**

This option is a solid cast piece with optimized structural geometry.

**Movement Range:** This option benefits the movement range since there is no need for clevis attachments that constrains the maximum raise of the upper arm.

**Manufacturing:** Must be outsourced to a supplier. From a stress failure point of view, a casting is of course far superior with the option of off center mass. The geometry with a linking tube is however complex where a simple two piece cast mould cannot be used. After casting, pin holes and taped holes needs to be machined. The large nature of the cast-part makes this difficult in Buhler’s facilities but very much possible.

**Cost:** A casting with more strategic placement of material can of course reduce material, and by so, also reduce cost in the long run. This component is however to be considered for small quantities where the cost of tooling for a casting greatly weigh in. A casting must therefore be designed never to fail. A great factor of safety must therefore be applied adding weight and material to the casting.

**Appearance:** Gives an optimized appearance that reflects a quality product.
**T-BEAM CASTING**

This option is a solid cast piece with optimized structural geometry with the constraint of still using clevis attachments for the lift rods and hydraulic cylinders. The basic idea for this option is to have a cast alternative to a solid cast plate.

**Material**

For a solid plate, only material mild steel ASTM A36 is to be considered. For a casting, the material is to meet with SAE’s standard for ductile cast, AUTOMOTIVE DUCTILE (NODULAR) IRON CASTINGS - SAE J434 JUN 86 [9]. The preferred properties are high yield strength within the ductile range. Considering this, the material selected is ductile cast iron grade D5506. This grade is in the upper range of tensile properties of the ductile options but still offers a considered sufficient elongation of 6%. Ductile iron D5506 is a widely used grade for Buhler applications and is the grade used for the current rock arm castings. Also considered was Ductile 100-70-03. This is a harder material with greater tensile properties but less elongation (material used for Buhler tractor “Genesis” Cat 3 hitch). Considering the low quantity of this casting, the cost of tooling is crucial to the part. Even if Grade D5506 has less tensile strength, it is considered as a safer option for a prototype. Grade D5506 is the more ductile out of the two which offers better prevention of stress concentration failure (plastic deformation in a ductile material allows stress to flow to a larger region around discontinuities). Also, there are no surrounding constraints for volume where a harder material could reduce size. Note that the safety of a successful prototype is of more importance then an optimized product for cost effectiveness (e.g. reduced material).

**Tubing**

Both rectangular and circular tubing was analyzed for implementation. Circular is naturally the optimal geometry in torsion where rectangular can give better features considering bending. Calculations however show that most loading to synchronize the two arms occurs in torsion making circular tubing is the natural choice. To minimize the tube shade from the driver view, the tube outer diameter is to be minimized. The outer diameter is minimized by thickening the wall-thickness where ¾ inch considered as thickest reasonable and is therefore the only thickness analyzed. Calculations are carried out in accordance with appendix A for a scenario of the upper arm level with the ground. The most loading occur at a raised positioning but spikes in torsion, where all load shifts on one side, can only occur during implementation in a lowered state.

<table>
<thead>
<tr>
<th>$d_C$ (outer diameter)</th>
<th>$\sigma_{dA,4}$ (MPa)</th>
<th>$\sigma_{dA,6}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5&quot; (127mm)</td>
<td>180</td>
<td>112</td>
</tr>
<tr>
<td>6&quot; (153mm)</td>
<td>120</td>
<td>75</td>
</tr>
<tr>
<td>7&quot; (178mm)</td>
<td>83</td>
<td>50</td>
</tr>
</tbody>
</table>

A 6inch tube is considered sufficient using a steel ASTM 1018 CD. This gives roughly the desired factor of safety around 3.

Considering section modules in torsion compared with the current design, the spline is approximated with a 3.5inch diameter solid cylinder. This gives a section modules in torsion of 8.5[in³] for the spline

---

4 SAE- Society of Automotive Engineering
where a 6in tube has a section modules of 29\(\text{in}^3\). Roughly considering the increase of load capacity by 50%, a 6in tube should be sufficient even though material properties can be assumed slightly weaker for the tube.

**Decision Matrix**

<table>
<thead>
<tr>
<th>Upper Arm</th>
<th>Options</th>
<th>Solid Plate</th>
<th>Two Plates</th>
<th>I-BEAM</th>
<th>T-BEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection Criteria</td>
<td>Weight Rating</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>Functionality</td>
<td>30%</td>
<td>2</td>
<td>0.6</td>
<td>4</td>
<td>1.2</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>15%</td>
<td>4</td>
<td>0.6</td>
<td>3</td>
<td>0.45</td>
</tr>
<tr>
<td>Cost</td>
<td>30%</td>
<td>5</td>
<td>1.5</td>
<td>4</td>
<td>1.2</td>
</tr>
<tr>
<td>Changeability</td>
<td>25%</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>Total Score</td>
<td></td>
<td>3.7</td>
<td>3.6</td>
<td>2.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Rank</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Continue?</td>
<td></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

A weldment of a solid plate is of course the most cost effective option where as the advantage with a casting lays in its endless options to optimize the structural geometry. A weldment can however be designed with less factor of safety where small modifications can be completed after testing of a prototype. It is concluded to use the solid plate option for a prototype.

**Stress Analysis of the solid plate option**

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{UA,Pin})</td>
<td>50.8</td>
<td>(T_{UA,Pin,\text{max}})</td>
<td>99</td>
<td>3.5</td>
</tr>
<tr>
<td>(r_{UA,ma})</td>
<td>33.0</td>
<td>(\sigma_{UA,Pin})</td>
<td>80</td>
<td>3.2</td>
</tr>
<tr>
<td>(t_{UA,ma})</td>
<td>50.8</td>
<td>(\sigma_{UA,\text{tearout}})</td>
<td>122</td>
<td>2.1</td>
</tr>
<tr>
<td>(d_{HC,Pin})</td>
<td>44.5</td>
<td>(\sigma_{UAHC,Pin})</td>
<td>94</td>
<td>2.7</td>
</tr>
<tr>
<td>(t_{UAHC,ma})</td>
<td>50.8</td>
<td>(\sigma_{UAHC,\text{tearout}})</td>
<td>76</td>
<td>3.3</td>
</tr>
<tr>
<td>(d_{LR,Pin})</td>
<td>44.5</td>
<td>(\sigma_{UA,LR,Pin})</td>
<td>102</td>
<td>2.5</td>
</tr>
<tr>
<td>(t_{UA,LR,ma})</td>
<td>50.8</td>
<td>(\sigma_{AA,UA,PL})</td>
<td>88</td>
<td>2.8</td>
</tr>
<tr>
<td>(r_{UA,LR,ma})</td>
<td>33.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculated stresses for assumed critical points give a tearout failure at UAP, the attachment to the rear frame, to be of most concern. This value is however calculated assuming worst case scenario for tearout. This is however not the case since the load is not strictly acting in the plane of the thinnest material. Therefore all values are considered satisfactory. FEA however result in some concern for the all load on one scenario. The opposite side experiences stresses peaking at 210MPa. This is however considered uncertain values where testing is to fully evaluate a failsafe product.

**Summary and conclusions**

The structures of the upper arm where modeled iteratively to reduce weight and stress using Inventor FEA and basic beam theory calculations (please refer to appendix C for load scenarios for FEA). It is finally concluded to use the option of solid plates linked together with a circular tube in a weldment. The plate thickness is set to 2inch and the tube to have an outer diameter of 6inch and a wall-thickness of \(\frac{3}{8}\)inch.
<table>
<thead>
<tr>
<th>Specifications:</th>
<th>UPPER ARM</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLICATION</td>
<td>PHASE C &amp; D 4WD TRACTORS</td>
</tr>
<tr>
<td>PART NUMBER</td>
<td>-</td>
</tr>
<tr>
<td>FINISH</td>
<td>BVI RED</td>
</tr>
<tr>
<td>PLATE MATERIAL</td>
<td>ASTM A36 PLATE NOMINAL SIZE 2&quot;</td>
</tr>
<tr>
<td>TUBE MATERIAL</td>
<td>ASTM 1018 CD NOMINAL SIZED 6x1/4&quot;</td>
</tr>
<tr>
<td>BORE FOR BUSHINS</td>
<td>2.5 +0.002 inch, 2.25 +0.002 inch</td>
</tr>
</tbody>
</table>
8.3 Hydraulic Cylinder

The Hydraulic Cylinder is only to be specified for key measurements where the final designed is conducted by the supplier. The required measurements to specify are the retracted length, the stroke, the piston diameter, the rod diameter and the maximum pressure. Also the design of the attachments at the top and bottom is to be specified for clearance.

The tractor can produce a pressure up to 3000psi. This sets a constraint of the hydraulic cylinder pressure. To assure safe use, the hydraulic system is equipped with a relief valve set at 3000psi. Test data however shows that the relief valve kicks in at 2895psi [3].

Neglecting friction and other energy losses in the linkage, the targeted pressure for max lift would be equal to the relief valve pressure. However due to the energy losses and uncertainties in the approximations, the calculated pressure for a max load of 20,000lb is however set slightly below the relief valve pressure, in the region 2750psi. For the resulting pressure in operation, the cylinder is mainly loaded in compression meaning the bottom surface of the piston is the key factor for regulating the resulting pressure. The top surface area of the piston rod side is neglected due to the minimal loading. The cylinder could be single acting with the current valve and hydraulic control setting. However, due to uncertainties of the systems weight as down force, the option of altering the hydraulic to double acting is desired. The retraction pressure is however minimal and the rod side of the piston can therefore be designed mainly for sufficient axial load strength (Euler column buckling).

For spikes of rapidly increased pressure, a cylinder specifying max working pressure around 3000psi is designed to withstand spikes roughly up to around the double, 6000psi. This is considered being sufficient safety before the relief valve kicks in or the load fades. Note that the relief valve is only for pump pressure. When the cylinder implement valve is in lock mode, the cylinder pressure holds no relief. Loads under normal working condition are however assumed to fall within the cylinder line of safety. Testing of the cylinder pressure during a 3G bump test described in section 2.3 is assumed to assure this.

The Hydraulic Cylinder suffers maximum load in the fully raised positioning of the hitch system. For this static scenario, the force $F_3$, acting on both cylinders, is calculated to 390kN. This result in the following data:

\[
F_3 = 390 \text{ kN} \Rightarrow \frac{F_3}{2} = 195 \text{ kN per cylinder}
\]

Table 8.5 shows resulting hydraulic pressure for varying diameter of the piston $d_{HC}$.

<table>
<thead>
<tr>
<th>$d_{HC}$</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>4&quot; (101.6mm)</td>
<td>3570 psi (24MPa)</td>
</tr>
<tr>
<td>4.5&quot; (114.3mm)</td>
<td>2760psi (19MPa)</td>
</tr>
<tr>
<td>5&quot; (127mm)</td>
<td>2240psi (15MPa)</td>
</tr>
</tbody>
</table>

In conclusion the Hydraulic Cylinder piston, $d_{HC}$, is set to a diameter of 4½inch. This to provide the desired working pressure around 2750psi for a maximum static load. The maximum potential load exerted from the cylinder is calculated for the maximum pressure of 3000psi:

\[
F_{HC,max}=212kN (48,000lb) \text{ per cylinder}
\]
**Rod Diameter**

The rod is to be designed to withstand buckling at maximum pressure.

<table>
<thead>
<tr>
<th>Euler formula for buckling in columns converted into minimum diameter for a circular moment of inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{critical}} = \frac{\pi^2 \cdot E \cdot I}{(K \cdot L)^2}$</td>
</tr>
<tr>
<td>$I_{\text{circular}} = \frac{\pi \cdot d^4}{64}$</td>
</tr>
<tr>
<td>$d_{\text{min}} = 4 \sqrt{\frac{P_{\text{max}} \cdot K^2 \cdot L^2 \cdot 64}{E \cdot \pi^3}}$</td>
</tr>
</tbody>
</table>

Constant $K$ for one end fixed and the other free of constraints $K=2$

Length of the rod outside of cylinder when fully extended $L < 350\text{mm}$

Maximum load on rod (3000psi) $F_{\text{max}}=212kN$

Module of Elasticity for assumed steel $E=210 \text{ GPa}$

Above gives a minimum rod diameter as follows: $d_{HC,\text{rod,min}}=32\text{mm}$

In conclusion the rod is to be designed to a minimum diameter of 2.25inch, roughly twice the critical diameter for buckling.

**Bottom Attachment**

The bottom attachment is to be designed for minimum material due to the surrounding constraints. The maximal force exerted from the cylinder on the bottom attachment is calculated from the maximum load capacity scenario. (The load from maximum pressure in the cylinder is slightly greater but the difference is neglected). For the specified loading scenario, Inventor FEA and basic calculations are conducted for design analysis. Note that calculations are only used as a reference for specifying basic geometry and key measurements for clearance. A supplier is to construct the complete design of specified data to assure secure operation.

The bottom hole is to be completed with a low friction bushing to reduce ware.

Mild Steel ASTM A36 is set as material and from there, the attachment is the iteratively modified to assure a safety factor of 3.

**Theoretical analysis in accordance with appendix A:**

| Table 8.6: Resulting stresses from set geometry |
|---|---|
| **Distance** | **Set Value [mm]** | **Analysis** | **Stress [MPa]** | **Factor of Safety** |
| $d_{HC,\text{Pin}}$ | 44.5 | $\sigma_{HC,ma,pin}$ | 87 | 2.9 |
| $t_{HC,ma}$ | 50.8 | | | |
Analysis of Stress data:

The safety factor of 2.9 is considered to be within the targeted factor of 3. A tearout is not considered to be an issue since the main load is in compression. FEA however give a minimum distance of 42mm from the bottom cylinder plate to the hole-center.

Upper Attachment

The upper attachment is to have a clevis structure consisting of a three plate weldment. For this geometry, the bottom plate inner side (facing the rear frame), is to have a chamfer sufficient to clear the Upper Arm at a fully raised positioning.

Mild Steel ASTM A36 is set as material and from there; the attachment is iteratively modified to assure a safety factor of 3.

Theoretical analysis in accordance with appendix A:

### Table 8.7: Resulting stresses for selected geometry

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{HC,cl})</td>
<td>44.5</td>
<td>(\sigma_{HC,cl})</td>
<td>87</td>
<td>2.9</td>
</tr>
<tr>
<td>(t_{HC,cl})</td>
<td>25.4</td>
<td>(\sigma_{A,HC,cl})</td>
<td>103</td>
<td>2.4</td>
</tr>
<tr>
<td>(L_{1,HC,cl})</td>
<td>56.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L_{2,HC,cl})</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L_{3,HC,cl})</td>
<td>76.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b_{HC,cl})</td>
<td>76.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h_{HC,cl})</td>
<td>31.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>kN</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(F_{max,HC,cl})</td>
<td>212</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analysis of Stress data:

Theoretical calculations give peaking stresses at 87 MPa at the pin joint which is comparable to the basic FEA which gives a stress pattern peaking around 90MPa for this area. This gives a factor of safety around 2.9 which is acceptable. At the weldment, stresses peak at 103 MPa. This is also considered acceptable due to the greater strength at welds which here is unrightfully neglected. Considering that there is a weld support, this is considered to be adequate.

The bottom plate is given dimensions as follows; the bottom plate is to have a thickness of 31.8mm with a 12x45° chamfer for clearance. A distance of 91mm from the bottom to the hole-center-line is considered to give sufficient clearance for the upper arm. For the spacing width, the clevis is to have a 3mm clearance at each side of the upper arm giving a total spacing of 57mm.

Furthermore, if misalignment becomes an issue, the Upper Arm is to be equipped with a swivel bushing. Also note that analysis is conducted with a cylinder rod diameter of 2.25inch. A smaller diameter of the rod will rapidly increase the stresses on the bottom plate.

*For basic calculations, please refer to Appendix A.*
*For load scenario for FEA, please refer to Appendix C.*
Prototype

The scope for the hydraulic cylinder is eventually altered to be preferred as a self manufactured product. The following key components are selected from existing parts for a prototype:

<table>
<thead>
<tr>
<th>PART NO</th>
<th>DESCRIPTION</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>23795</td>
<td>TUBE 4.5” DIA</td>
<td>Needs to be shortened.</td>
</tr>
<tr>
<td>112561</td>
<td>PISTON 4.5” DIA</td>
<td>Thickness: 1.13”</td>
</tr>
<tr>
<td>112563</td>
<td>SHAFT 2.25”</td>
<td>Cylinder Rod. Needs to be shortened. Piston nut: 1.12”</td>
</tr>
<tr>
<td>24888</td>
<td>4.50 DIA HEADPLATE</td>
<td>Thickness: 2.75”</td>
</tr>
<tr>
<td>7549010204</td>
<td>CLEVIS 2.00 LIFT CYLINDER ROD</td>
<td>Flame cut clevis.</td>
</tr>
</tbody>
</table>

**Stress analysis of Upper Clevis Attachment 7549010204**

*Please refer to Appendix A for brief description of hand calculations*

Solid piece; material mild steel ASTM A36.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{HC,Pin}$</td>
<td>44.5</td>
<td>$\sigma_{HC,Pin}$</td>
<td>87</td>
<td>2.9</td>
</tr>
<tr>
<td>$t_{HC,cl}$</td>
<td>25.4</td>
<td>$\sigma_{A,HC,cl}$</td>
<td>72</td>
<td>3.5</td>
</tr>
<tr>
<td>$L_{1,HC,cl}$</td>
<td>76.2</td>
<td>$F_{max,HC,cl}$</td>
<td>212</td>
<td></td>
</tr>
<tr>
<td>$L_{2,HC,cl}$</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{3,HC,cl}$</td>
<td>57.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{HC,cl}$</td>
<td>76.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{HC,cl}$</td>
<td>50.8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thickness of bottom could possibly be stretched thinner to increase the stroke but the part is initially concluded to be used as is.

**Head Size**

The head is used for a cylinder with greatly larger length and stroke which causes it to be thicker than anticipated for this cylinder. The head thickness could be shaved a bit but this would mean redesigning the grove pattern for seal parts. It is concluded to use the part as is and possible modifications to gain stroke is initially referred to the upper clevis attachment.

**Fixed length and Stroke**

Fixed length is calculated to 323mm. Iterative modeling gives a stroke of 280mm to give sufficient movement range (note: noticeable modifications are made to the linkage components and pivoting points). The cylinder results in a retracted length of 603mm and an extended length of 883mm.
Conclusions in Summary

The hydraulic cylinder is designed to have a piston diameter of 4.5 inch, rod diameter of 2.25 inch, tube wall-thickness of \( \frac{3}{16} \) inch and a bottom plate of \( \frac{1}{2} \) inch. The bottom end consists of a solid 2 inch plate completed with a low friction bushing and the top with a three plate clevis weldment. The cylinder was designed using an assumed fixed length of 300 mm and for this; a stroke of 295 mm is preferred. The extended length of 890 mm is however key and to be targeted. From there, the retracted length should be optimized by maximizing the stroke.

As mentioned above, for a prototype Buhler Versatile is considering to manufacture an in house cylinder using excising and modified parts. For this option, standardized measurements will be slightly compromised for prototype.

Proposed

<table>
<thead>
<tr>
<th>Specifications: HYDRAULIC CYLINDER, CATEGORY IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLICATION</td>
</tr>
<tr>
<td>PART NUMBER</td>
</tr>
<tr>
<td>FINISH</td>
</tr>
<tr>
<td>MAX PRESSURE</td>
</tr>
<tr>
<td>PISTION DIAMETER</td>
</tr>
<tr>
<td>ROD DIAMETER</td>
</tr>
<tr>
<td>MOVEMENT RANGE</td>
</tr>
<tr>
<td>BOTTOM END</td>
</tr>
<tr>
<td>TOP END</td>
</tr>
</tbody>
</table>
8.4 Lower Link

The lower links are preferably designed similar to the current lower links using steel bar stock. Two cross-section sizes, 2.5by5inch and 2by6inch, are to be considered and evaluated for implementation. Also a cast option is analyzed.

Material

Considering the high stresses exerted on the lower links, a stronger material then preferred mild steel A36 is required. It is concluded to use steel AISI 4140 or material of similar nature. This is a material of good fatigue and impact resistance. Also, it has great hardenability for potential future strengthening of the design. For the current design, the bore of the lift rod attachment is induction hardens and also the center section, in between the lift rod and hitch points, is heat treated. At first a prototype is however to be constructed of annealed steel where the options of hardening the material is to be considered after testing.

For a Cast Option, cast steel ASTM 4140 is to be considered.

Geometry

To increase the implement stability, ASAE standards specify to have lower links that converge towards a point within a specific range. In order to achieve this, the lower links cannot maintain its simple straight geometry. To maintain parallel geometry at both ends, bends must be applied. Three different options are considered where one only has two bends at the attachment points and one option also has one bend midway of the link to reduce tilt of the lower link due to alignment with the lift rod. A cast option can of course be designed for optimal geometry.

Cast Option

There are numerous advantages by going with a cast option for the lower link. The link cross-section can be continually altered to match the bending load diagram where the peak of the stresses exerted is located at the lift rod attachment. Increasing the cross section height at this point not only reduces stresses, but the hole for the lift rod attachment can be located above the center axis of the lower link. For this, the lift rod load acts at an angle where the higher it is located, the closer the load acts to the hitch points. Aligning these load point is an advantage from a load point of view but also calls for two attachments for the lift rods to keep left and right link within one casting (unless the attachment is put on both sides of the link).
Stress Analysis

Please refer to Appendix A for variable description and calculations.
For load scenario for FEA, please refer to Appendix C.

Analysis bar 2by6” cross section

Table 8.8: Set values and the resulting stresses for bar 2by6”

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_{LLP,Pin}</td>
<td>50.8</td>
<td>t_{LLP,Pin,max}</td>
<td>202</td>
<td>1.7</td>
<td>Pull</td>
</tr>
<tr>
<td>d_{LR,Pin}</td>
<td>44.5</td>
<td>t_{LLP,Pin}</td>
<td>192</td>
<td>2.2</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>42</td>
<td>σ_{LLP,Pin}</td>
<td>256</td>
<td>1.6</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>38.1</td>
<td>σ_{LLP,tearout}</td>
<td>283</td>
<td>1.5</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLR}</td>
<td>50.8</td>
<td>σ_{LLR,Pin}</td>
<td>79</td>
<td>5.3</td>
<td>Lift</td>
</tr>
<tr>
<td>D_{LL,LR}</td>
<td>150</td>
<td>σ_{xx,A,bar}</td>
<td>133</td>
<td>3.2</td>
<td>Lift</td>
</tr>
<tr>
<td>d_{LL,bar}</td>
<td>50.8</td>
<td>σ_{xx,B,bar}</td>
<td>-192</td>
<td>2.2</td>
<td>Lift</td>
</tr>
<tr>
<td>b_{LL,bar}</td>
<td>150</td>
<td>σ_{xx,A,bar,C}</td>
<td>-58</td>
<td>7.3</td>
<td>Pull</td>
</tr>
<tr>
<td></td>
<td></td>
<td>σ_{xx,B,bar,C}</td>
<td>192</td>
<td>2.2</td>
<td>Pull</td>
</tr>
</tbody>
</table>

Analysis bar 2½by5” cross section

Table 8.9: Set values and the resulting stresses for bar 2½by5”

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_{LLP,Pin}</td>
<td>50.8</td>
<td>t_{LLP,Pin,max}</td>
<td>202</td>
<td>1.7</td>
<td>Pull</td>
</tr>
<tr>
<td>d_{LR,Pin}</td>
<td>44.5</td>
<td>t_{LLP,Pin}</td>
<td>192</td>
<td>2.2</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>42</td>
<td>σ_{LLP,Pin}</td>
<td>256</td>
<td>1.6</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>38.1</td>
<td>σ_{LLP,ma,max}</td>
<td>283</td>
<td>1.5</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLR}</td>
<td>63.5</td>
<td>σ_{LLR,Pin}</td>
<td>63</td>
<td>6.6</td>
<td>Lift</td>
</tr>
<tr>
<td>D_{LL,LR}</td>
<td>127</td>
<td>σ_{xx,A,bar}</td>
<td>152</td>
<td>2.8</td>
<td>Lift</td>
</tr>
<tr>
<td>d_{LL,bar}</td>
<td>63.5</td>
<td>σ_{xx,B,bar}</td>
<td>-212</td>
<td>2.0</td>
<td>Lift</td>
</tr>
<tr>
<td>b_{LL,bar}</td>
<td>127</td>
<td>σ_{xx,A,bar,C}</td>
<td>-73</td>
<td>5.7</td>
<td>Pull</td>
</tr>
<tr>
<td></td>
<td></td>
<td>σ_{xx,B,bar,C}</td>
<td>208</td>
<td>2.0</td>
<td>Pull</td>
</tr>
</tbody>
</table>

Analysis Casting

Table 8.10: Set values and the resulting stresses casting

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_{LLP,Pin}</td>
<td>50.8</td>
<td>t_{LLP,Pin,max}</td>
<td>202</td>
<td>1.7</td>
<td>Pull</td>
</tr>
<tr>
<td>d_{LR,Pin}</td>
<td>44.5</td>
<td>t_{LLP,Pin}</td>
<td>192</td>
<td>2.2</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>42</td>
<td>σ_{LLP,Pin}</td>
<td>256</td>
<td>1.6</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLP,ma}</td>
<td>38.1</td>
<td>σ_{LLP,ma,max}</td>
<td>283</td>
<td>1.5</td>
<td>Pull</td>
</tr>
<tr>
<td>t_{LLR}</td>
<td>95.3</td>
<td>σ_{LLR,Pin}</td>
<td>100</td>
<td>4.2</td>
<td>Lift</td>
</tr>
<tr>
<td>D_{LL,LR}</td>
<td>40</td>
<td>σ_{LLR,tearout}</td>
<td>84</td>
<td>5.0</td>
<td>Lift</td>
</tr>
<tr>
<td>d_{LL,bar}</td>
<td>80</td>
<td>σ_{xx,A,bar}</td>
<td>155</td>
<td>2.7</td>
<td>Lift</td>
</tr>
<tr>
<td>b_{LL,bar}</td>
<td>15</td>
<td>σ_{xx,B,bar}</td>
<td>-161</td>
<td>2.6</td>
<td>Lift</td>
</tr>
<tr>
<td>a_{2,LL,bar}</td>
<td>70</td>
<td>σ_{xx,A,bar,C}</td>
<td>-85</td>
<td>4.9</td>
<td>Pull</td>
</tr>
<tr>
<td>a_{3,LL,bar}</td>
<td>45</td>
<td>σ_{xx,B,bar,C}</td>
<td>158</td>
<td>2.7</td>
<td>Pull</td>
</tr>
</tbody>
</table>
Stress Analysis in summary

The stress analysis gives the bar cross-section of greater height, 2by6, as a better option. The bending curvature of the link is however neglected in the hand calculated values where the increase of width would be beneficial. The FEA analysis gives the width to give better stress distribution along the link, but the critical point is still at the lift rod cross section where the height of the bar reduces the stresses the most. Cast is naturally the superior for the geometry to reduce stresses, but is also less cost effective and only to be considered as an alternative option.

The safety factor of the lower link is significantly less than the other components. All stresses with safety factor less than 2.5 is however calculated from a pulling scenario where most load is exerted on the lower link (calculated for all load shifted to one link scenario). This is a very extreme scenario which is unlikely to occur. It is decided to use less factor of safety where extensive field testing is to assure a failsafe product.

The tearout stress at LLP is the weakest point which is to be considered for strengthening. This link is designed to have the same geometry at each end where the hitch point clearance constrains the material thickness.

Sway Block

The original idea was to use a similar concept as the current design were the lower links rubs of blocks to restrict its lateral movement in a plane horizontal to the ground. This is the most common solution for the North American market where the European market more often uses stabilizing links. The stabilizing links have the same function as the lift rod where it can be set for free float within a restricted range or locked at a fixed length.

For implementation of rubbing blocks interchangeable between two categories, there is only roughly 20mm to work with for the blocks on cat 4N. It interferes with the drawbar cage which could be moved at the cost of less adjustment of the drawbar or by moving the lower links and by so, sacrificing convergence for stability. Stabilizers however are free of these constraints. It’s overall a better solution that gives less stress distribution on the lower links and better stability. Rubbing blocks are a more cost effective option than a stabilizer but considering the heavy duty nature of this application it is concluded to use stabilizers.

For attachment of the stabilizer to the lower link, there is to be two settings for cat 4 and cat 4N. The attachment is to be located as close to the lift rod as possible within sufficient clearance.

Desired length and movement range to specify giving a rough lateral free float of 130mm at the hitch points: 403mm +/- 36mm

Conclusions in Summary

For a first prototype, it is concluded to go with a 2by6 bar with two forgings welded on to each end at an angle. The prototype is to be tested and from there the design can be strengthened using heat treatment for critical points. A cast option is a better option but is considered to be too costly for this application. It is to be equipped with cat 4 swivel balls at both ends and holes for attachment of stabilizer bracket.
## Proposed Specifications: Lower Link, CATEGORY IV

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>APPLICATION</strong></td>
<td>PHASE C &amp; D 4WD TRACTORS</td>
</tr>
<tr>
<td><strong>PART NUMBER</strong></td>
<td>86037128</td>
</tr>
<tr>
<td><strong>FINISH</strong></td>
<td>BVI RED</td>
</tr>
<tr>
<td><strong>MATERIAL</strong></td>
<td>SAE 4140 STEEL (2x6” NOMINAL SIZE)</td>
</tr>
<tr>
<td><strong>LINK ENDS</strong></td>
<td>EQUIPPED WITH SWIVEL BUSHINGS, PIN SIZE 2” DIA</td>
</tr>
<tr>
<td><strong>LINK END WELD</strong></td>
<td>BEVELLED WELD TYP</td>
</tr>
<tr>
<td><strong>LOAD RATINGS AT LINK END</strong></td>
<td>a) 20,000 LBS LIFT</td>
</tr>
<tr>
<td></td>
<td>b) 20,000 LBS PULL</td>
</tr>
<tr>
<td></td>
<td>c) COMBINATION LOAD OF 10,000 LBS LIFT AND 10,000 PULL SIMULTANEOUSLY</td>
</tr>
</tbody>
</table>
8.5 Lift Rod

The lift rod has been the main concern for failure on the current 3pt system and needs to be significantly strengthened. It is preferred to go away from the current ball joint at the upper arm to a clevis attachment. The lift rod is to be designed with basic specifications for quotations from suppliers. The quotes are then to be compared with varying shelf products and design proposals offered by suppliers.

Free Float

The lift rod is to be able to free float a minimum of 75mm at the hitch points. It is concluded to use a T-Forging with a locking pin rather than the previous slotted hole. A free float distance of 70mm at the lift rod is to give sufficient torsional free float at the hitch points. The free float locking pin is to be the weakest link of the assembly. It is concluded that a pin diameter around safety factor of 2 should be ideal.

Threads

The current threaded rods are 1.5inch in diameter with 4½ threads/in. Considering the high stresses and vibrations exerted on the lift rod, it is concluded to use a more fine pitched thread which is more commonly used in vibrating environments such as automotive vehicles. More common threads for this kind of application are 8 threads/in or metric pitch of 3mm. Also, rolled threads offer much stronger resistance against stripping of the threads and are therefore preferred. One side is to be threaded with right-hand threads (RH) and the opposite with left hand threads (LH). This results in simultaneous extension or retraction of the threaded rods by turning the adjustment tube. The attachment of the threaded rod to the clevis casting is concluded to be threaded where the rotation locks in place by a roll pin.

Material selection

The lift rod dimensions is to be specified as a reference where the main specification to consider for a supplier is the rated load capacity. The threaded rod with T-Forging is mainly to be designed by supplier to match specified load capacity with preferred manufacturing. The opposite threaded rod is concluded to be ASTM 1045 CD. The upper and lower attachments are concluded to be clevis castings and also the linking tube is concluded to be a casting. Considering the stress distribution on the clevis, the stress distribution on the threaded linking points, and various grades for this kind of application, it is considered sufficient to use Buhler Versatile commonly preferred grade ductile cast iron grade SAE 5506 for all the castings.
Stress Analysis

Please refer to Appendix A for calculations.
For FEA load scenarios, please refer to Appendix C.

Table 8.11: Set values and the resulting stresses for the lift rod

<table>
<thead>
<tr>
<th></th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{LR,Pin}$</td>
<td>44.5</td>
<td>$\tau_{LR,Pin,max}$</td>
<td>102</td>
<td>3.4</td>
</tr>
<tr>
<td>$d_{LR,ffpin}$</td>
<td>31.8</td>
<td>$\tau_{LR,ffpin,max}$</td>
<td>155</td>
<td>2.25</td>
</tr>
<tr>
<td>$t_{LR,cl}$</td>
<td>25.4</td>
<td>$\sigma_{LR,cl,max}$</td>
<td>128</td>
<td>3.1</td>
</tr>
<tr>
<td>$p_{LR}$</td>
<td>3.0</td>
<td>$\sigma_{LR,cl,pin}$</td>
<td>70</td>
<td>5.7</td>
</tr>
<tr>
<td>$d_{LR,RL}$</td>
<td>80.0</td>
<td>$\sigma_{LR,cl,tearout}$</td>
<td>123</td>
<td>3.3</td>
</tr>
<tr>
<td>$d_{LR}$</td>
<td>50.0</td>
<td>$\tau_{LR}$</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>$F_{2,xyz,max}$</td>
<td>160</td>
<td>$\tau_{LR,nut,thread}$</td>
<td>450</td>
<td>per thread</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_{LR,thread}$</td>
<td>380</td>
<td>per thread</td>
</tr>
</tbody>
</table>

Stress analysis in summary

The lift rod is evaluated for its maximum loading scenario where the linkage is fully raised. The maximum load in tension is calculated to 160kN which sets the load capacity to specify. For the threads, even if the shear stress is greater for the threads on the rod, the “nut” threads (the castings) are mainly to be considered since this is the weakest material and is assumed to start yield first. Around 400MPa per thread gives a standard thread distance equal to the rod diameter to be more than sufficient, Norton (2000) however recommends a thread distance of 1.5 times the diameter of the rod for threads in cast iron. The specified material, SAE D5506, is however a ductile cast iron where it is realistic to assume more simultaneous yield of all thread at once. Therefore, all thread linking points of the lift rod is concluded to have minimum 60mm of threads in contact.

For the rod diameter, targeting stresses around 90MPa gives a diameter of 50mm. The threaded rod is therefore to be specified as M50x3 thread (Alternative: 2-8-UN).

The free float pin is only evaluated for compression loads from a pull scenario and is designed to be the weakest link. A diameter of 1.25inch giving a safety factor of 2.25 is considered sufficient for the pulling scenario described in Appendix A.

The upper clevis, connecting to the upper arm, is set to have slightly thicker side plate since its bending load is neglected in the calculations.

Length range

The lift rod lengths is set by the standardized hitch point positioning for transport height (fully raised) and lower hitch point height (fully lowered). Measurements in modeling gives retracted length of 1085mm and extended length of 1287mm to be sufficient for basic scenario with lower link of bar stock (with hole for LR in the center plane of the link). For a cast lower link, the positioning of the link would be altered and the length should be shortened accordingly.

Accessories

Turnbuckle: The lift rod is to be equipped with a turnbuckle for extending or retracting the rod by turning the adjustment tube. The turnbuckle is to lock the tube in place when not in use. It concluded to have a hex geometry of the tube linkage that fits with square turnbuckle geometry. The turnbuckle is then to be attached to the upper clevis for locking the tube.
**Lubrication:** The adjustment tube linkage is to be equipped with grease fittings at both ends to lube the threads. For the pivoting points, the pin is to be fixed at both ends of the lift rod. The mid pivoting point is to be equipped with a grease fitting. (The rotation about this pivot is minimal but the same part is however implemented for the upper link with greater rotation). The fitting is to be placed in the same plane as the other holes to reduce machine operations.

**Thread locking device in Linking Tube:** To assure not extending the link too far, the threaded rod is to be equipped with a slotted pin once the rod is threaded into the tube. For this operation, the tube must be equipped with two holes that are to be plugged after this procedure.

**Proposed Specifications:**

<table>
<thead>
<tr>
<th>Specifications: Lift Link, CATEGORY IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLICATION</td>
</tr>
<tr>
<td>PART NUMBER</td>
</tr>
<tr>
<td>SIMILAR PART</td>
</tr>
<tr>
<td>SURFACE TREATMENT</td>
</tr>
<tr>
<td>OTHER PARTS</td>
</tr>
<tr>
<td>PINISH</td>
</tr>
<tr>
<td>THREADED ROD</td>
</tr>
<tr>
<td>THREADS</td>
</tr>
<tr>
<td>CASTING GRADE</td>
</tr>
<tr>
<td>PIN DIAMETER AT ENDS</td>
</tr>
<tr>
<td>FREE FLOAT PIN DIAMETER</td>
</tr>
<tr>
<td>FREE FLOAT RANGE</td>
</tr>
<tr>
<td>RATED LOAD CAPACITY</td>
</tr>
</tbody>
</table>
8.6 Upper Link

For the Upper Link, it is preferred to go away from the current ball joints at each end. The lift rod is to be designed with basic specifications for quotations from suppliers. The quotes are then compared with varying shelf products and design proposals offered by suppliers. Components of the lift link and the lift rod are preferred to be shared, especially the castings.

Threads

The Upper Link is to have the same adjustment tube as the lift rods. Please refer to section 8.5, analysis of threads on Lift Rod.

Material selection

The Upper Link is to have the same components as the Lift Rods for clevis attachments and adjustment tube. Please refer to section 8.5, analysis of material selection for Lift Rod. The threaded rods are concluded to be steel forgings.

Stress Analysis

Please refer to Appendix A for calculations.

For FEA load scenarios, please refer to Appendix C.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{UL,Pin}$</td>
<td>44.5</td>
<td>$\tau_{UL,Pin,max}$</td>
<td>102</td>
<td>3.4</td>
</tr>
<tr>
<td>$t_{UL,ma}$</td>
<td>50.0</td>
<td>$\sigma_{UL,ma,max}$</td>
<td>129</td>
<td>3.1</td>
</tr>
<tr>
<td>$r_{UL,ma}$</td>
<td>25.4</td>
<td>$\sigma_{UL,ma,Pin}$</td>
<td>70</td>
<td>5.7</td>
</tr>
<tr>
<td>$\rho_{UL}$</td>
<td>3.0</td>
<td>$\sigma_{UL,ma,tearout}$</td>
<td>123</td>
<td>3.3</td>
</tr>
<tr>
<td>$d_{UL,RL}$</td>
<td>80.0</td>
<td>$\sigma_{UL}$</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>$d_{UL}$</td>
<td>50.0</td>
<td>$\sigma_{UL,RL}$</td>
<td>51</td>
<td>7.7</td>
</tr>
<tr>
<td>Force</td>
<td>[kN]</td>
<td>$\tau_{UL,thread}$</td>
<td>436</td>
<td>per thread</td>
</tr>
<tr>
<td>$F_{UL,C}$</td>
<td>-156</td>
<td>$\tau_{UL,nut,thread}$</td>
<td>378</td>
<td>per thread</td>
</tr>
</tbody>
</table>

Stress analysis in summary

The upper link is evaluated for its maximum loading scenario where an implement is being pulled with an up-force distributing zero weight on the rear wheels. For this scenario, the maximum load on the upper link is calculated to roughly 160kN in compression which sets the load capacity to specify. The load ratio lift rod vs. upper link is very similar. Therefore, it is concluded to use the same dimensions and casting components specified for the lift rod.

Accessories

Turnbuckle: The Upper Link is to be equipped with the same turnbuckle as described in section 8.5.

Lubrication: The pivoting link between the threaded rod forging and the clevis casting is to be neglected for lubrication due to minimal rotation. The clevis is however to be equipped with grease fittings for frame attachment and hitch point pivot.

Thread locking device in Adjustment Tube: Please refer to section 8.5 lift rod for same part.
### Proposed

<table>
<thead>
<tr>
<th>Specifications: Top Link, CATEGORY IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLICATION</td>
</tr>
<tr>
<td>PART NUMBER</td>
</tr>
<tr>
<td>SIMILAR PART</td>
</tr>
<tr>
<td>SURFACE TREATMENT</td>
</tr>
<tr>
<td>OTHER PARTS</td>
</tr>
<tr>
<td>FINISH</td>
</tr>
<tr>
<td>THREADED ROD</td>
</tr>
<tr>
<td>THREADS</td>
</tr>
<tr>
<td>CASTING GRADE</td>
</tr>
<tr>
<td>PIN DIAMETER AT ENDS</td>
</tr>
<tr>
<td>RATED LOAD CAPACITY</td>
</tr>
</tbody>
</table>
9 Design of Attachment Assembly

The attachment points for the linkage components are desired to be linked together into one big weldment.

9.1 Upper Link Point

For a prototype it is concluded to design for the option with PTO. The ULP attachment bracket is to be bolted on the main attachment assembly to be able to install/uninstall the PTO-box. It is concluded to use similar geometry as the current design.

Stress Analysis

Please refer to Appendix A for calculations

<table>
<thead>
<tr>
<th>Distance</th>
<th>Set Value [mm]</th>
<th>Analysis</th>
<th>Stress [MPa]</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_{1,ULP}</td>
<td>100</td>
<td>σ_{eff,ULP,A}</td>
<td>54</td>
<td>-</td>
</tr>
<tr>
<td>L_{2,ULP}</td>
<td>320</td>
<td>σ_{eff,ULP,B}</td>
<td>53</td>
<td>-</td>
</tr>
<tr>
<td>L_{3,ULP}</td>
<td>60</td>
<td>σ_{eff,ULP,C}</td>
<td>44</td>
<td>-</td>
</tr>
<tr>
<td>L_{4,ULP}</td>
<td>260</td>
<td>σ_{eff,ULP,D}</td>
<td>69</td>
<td>3.6</td>
</tr>
<tr>
<td>a_{1,ULP}</td>
<td>260</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b_{1,ULP}</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a_{2,ULP}</td>
<td>117</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b_{2,ULP}</td>
<td>19.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a_{3,ULP}</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b_{3,ULP}</td>
<td>19.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>[kN]</td>
<td>F_{ULC}</td>
<td>-156</td>
<td></td>
</tr>
</tbody>
</table>

The safety factor of 3.6 is set greater than 3. This is however considered reasonable when taking into account the neglected side load of upper link when implement has shifted to the side. Also neglected is the mid section bending and the pulling scenario is, as mentioned, a very rough approximation.

Bolts

It is concluded to use 8 bolts M20x2.5 cl.10.9. This is greatly oversized due to difficulty of estimating bending loads acting on the bolts due bending of bracket plate.

9.2 Upper Arm Point

For the upper arm attachment, no thorough stress calculations are conducted. Plates are chosen to handle acting sheer from pin with ease and the plates are gusseted for extra stability. Further analysis of the attachment weldment is to be conducted after testing.

Sensor

The upper arm point is to be equipped with a sensor to digitally control the linkage by readings of the upper arm angle. For the current design the sensor is mounted on to the upper arm at the center pivot. This is however not possible for the new design due to the fixed pins. Instead the sensor is to be positioned below the upper arm attachment point and connect to the upper arm via a sub linkage.
9.3 Lower Link Point

For the upper arm attachment, no thorough stress calculations are conducted. Plates are chosen to handle acting sheer from pin with ease. Further analysis of the attachment weldment is to be conducted after testing.

The lower link point is positioned beneath the rear frame and must therefore be linked with the outer stabilizer attachment for extra support. This is carried out by extending the lower link pin through the stabilizer attachment plate. This will not only give the two attachments extra support but also align them for equal constraints of the lower link throughout the lift cycle. The pin is extended through a tubing to increase its bending strength.

9.4 Drawbar cage

The drawbar cage cross-member that holds the drawbar is to be located to position the drawbar 22” above the ground [2]. For the drawbar angular settings, the options are to be reduced to three positions compared with a non three-point hitch installation.

The side plates are to be located as far out as possible. This locates them closer to the outer frame plates which are significantly stronger than the mid plate from a pull load point of view.
10 Proposed Design

Shown in figure 10.1 is the proposed design that meets with ASAE standard for category 4 and is interchangeable with category 4N. Targeted lift capacity is slightly greater than 20,000lb and the pulling capacity is set to match a tractor working in the region of 500HP. The complete design meets with ASEA standards specified in section “1.4 Target Specifications” with the exception of slightly compromised convergence for category 4N. The attachment assembly is joined together in one big weldment where the upper link attachment bracket is a bolt on for installation of a PTO-box. For lateral constraint it is concluded to use stabilizers. The stabilizer attachment assembly bolts to the side of the rear frame and is pined to the main attachment assembly for both alignment of the links and giving the lower link point extra support for pulling loads. For interchangeability between cat 4 and cat 4N, the lower link is equipped with two settings for the stabilizer attachment bracket. Also the upper link attachment is equipped with two settings for the upper link depending on the preferred working conditions of the implement. The upper setting is for level mast pitch at transport height and the lower setting for optimal tail clearance of an implement at transport height.

The overall design meets with the overall objective of simplicity and cost effectiveness. The complete attachment assembly consists only of standard shelf plates and tubes joined in one big weldment. The upper arm consists of a simple plates and tube weldment which replaces the costly rock arms and spline system. Stabilizers have been implemented instead of the previous sway block system mostly due to that stabilizers offer better support to the lower links which is considered key for this heavy duty application. Also, the use of stabilizers offers the possibility to have the design interchangeable between cat 4 and cat 4N and keep three settings for the drawbar. This was not
possible with sway blocks.
For the linkage components upper link and lift rod, a lot of shelf products have been evaluated for feasibility but none matched the required specifications. Instead, a complete set of links was designed and drawn up for quotations.

In conclusion the presented design meets with the targeted simplicity that characterise the Buhler Versatile brand. The desired cost effectiveness had to be somewhat compromised for the linkage components but reduction of the lift capacity for more cost effective shelf products was not an option. Considering the design as being a market leader in load capacity, it is surly to reflect positive on Buhler Versatile as a brand on the agricultural market.
REFERENCES

[2] ASAE STANDARD. ASAE S482 FEB04 – Drawbars-Agricultural Wheel Tractors
[3] SUMMARY OF OECD TEST 1942 NEBRASKA SUMMARY 351A BUHLER VERSATILE 2360 DIESEL 12 SPEED.
[6] ASAE STANDARD, ASAE S278.7 JUL03 (ISO 11001-1 1993) – Agricultural wheeled tractors and implements – Three-point hitch couplers
[9] SAE STANDARD, SAE J434 JUN 86 - AUTOMOTIVE DUCTILE (NODULAR) IRON CASTINGS
A.1 Geometry Equations and Constraints

The linkage-system is approximated to a 2D scenario where the components positions are calculated for the following constraints.

**Constraints for Upper Arm and Hydraulic Cylinder**

\begin{align*}
\text{Eq1} & : & H_{C_{px}} + H_{C_{vx}} &= U_{A_{px}} + UAH_{C_{vx}} \\
\text{Eq2} & : & H_{C_{py}} + H_{C_{vy}} &= U_{A_{py}} + UAH_{C_{vy}} \\
\text{Eq3} & : & H_{C_{vx}}^2 + H_{C_{vy}}^2 &= H_{C_{length}}^2 \\
\text{Eq4} & : & UAH_{C_{vx}}^2 + UAH_{C_{vy}}^2 &= UAH_{C_{length}}^2 \\
\text{Eq5} & : & UALR_{v} = \frac{UAL_{length}}{UAHC_{v}} \cdot UAH_{C_{v}} \\
\end{align*}

**Constraints for Lower Link, Lift Rod and Upper Arm**

\begin{align*}
\text{Eq6} & : & L_{L_{px}} + LLL_{vx} + L_{R_{vx}} &= U_{A_{px}} + UAL_{R_{vy}} \\
\text{Eq7} & : & L_{L_{py}} + LLL_{vy} + L_{R_{vy}} &= U_{A_{py}} + UAL_{R_{vy}} \\
\text{Eq8} & : & LLL_{vx}^2 + LLL_{vy}^2 &= LLLR_{length}^2 \\
\text{Eq9} & : & L_{R_{vx}}^2 + L_{R_{vy}}^2 &= LR_{length}^2 \\
\text{Eq10} & : & LL_{v} = \frac{LL_{length}}{LL_{length}} \cdot LLL_{v} \\
\end{align*}

**Constraints for Lower Link, Mast and Upper Link**

\begin{align*}
\text{Eq11} & : & LL_{px} + LL_{vx} + M_{vx} &= UL_{px} + UL_{vx} \\
\text{Eq12} & : & LL_{py} + LL_{vy} + M_{vy} &= UL_{py} + UL_{vy} \\
\text{Eq13} & : & M_{vx}^2 + M_{vy}^2 &= M_{length}^2 \\
\text{Eq14} & : & UL_{vx}^2 + UL_{vy}^2 &= UL_{length}^2 \\
\end{align*}

Equation 1-14 gives the resulting output variables.

Note: The equations are of second degree and results in two solutions.

- Eq1-Eq4: The max value of $H_{C_{vx}}$ is selected
- Eq6-Eq9: The min value of $L_{R_{vy}}$ is selected
- Eq11-Eq14: The max value of $UL_{vy}$ is selected
Angles of the components in a 2D plane

Lower Link \[ \varphi_1 = \arctan \left( \frac{LL_{hp} - LL_{py}}{LL_{hp} - LL_{px}} \right) \]

Upper Link \[ \varphi_2 = \arctan \left( \frac{UL_{hp} - UL_{py}}{UL_{hp} - UL_{px}} \right) \]

Hydraulic Cylinder \[ \varphi_3 = \arctan \left( \frac{UAHC_{py} - HC_{py}}{UAHC_{px} - HC_{px}} \right) \]

Upper Arm \[ \varphi_4 = \arctan \left( \frac{UALR_{py} - UA_{py}}{UALR_{px} - UA_{px}} \right) \]

Lift Rod \[ \alpha = \arctan \left( \frac{UALR_{px} - LLLR_{py}}{LLL_{px} - UALR_{px}} \right) \]

Mast \[ \theta = \arctan \left( \frac{LL_{hp} - UL_{hp}}{UL_{hp} - LL_{hp}} \right) \]
A.2 Forces and Free Body Diagrams
The resulting forces in section A.2.1 are calculated from the ASAE Standard test scenario [1] with applied load $P$. Stress analysis is then conducted with the extreme loading scenario of all weight shifted to one side. The distances and load $P$ are given as input data and the forces as output data.

In section A.2.2 resulting forces is calculated for a pulling scenario with an implement in use.

Figure A.4: Distances to pivot points
A.2.1 ASAE Lifting Scenario

Free Body Diagram Mast

![Free Body Diagram Mast]

Figure A.5: Mast frame with applied load P

\[ -F_1 \cdot \sin(\varphi_2) + F_y = P = 0 \quad \text{Eq15} \]
\[ -F_1 \cdot \cos(\varphi_2) + F_x = 0 \quad \text{Eq17} \]
\[ F_1 \cdot \cos(\varphi_2) \cdot L_2 \cdot \cos(\theta) + F_1 \cdot \sin(\varphi_2) \cdot L_5 \cdot \sin(\theta) - P \cdot L_4 \cdot \cos(\theta) = 0 \quad \text{Eq18} \]

Free Body Diagram Lower Link and Lift Rod

The forces are calculated for an extreme scenario where the full load is shifted to one side.

![Free Body Diagram Lower Link and Lift Rod]

Figure A.6: Lower Link

\[ -L_y + F_2 \cdot \sin(\alpha) - F_y = 0 \quad \text{Eq19} \]
\[ L_x - F_2 \cdot \cos(\alpha) - F_x = 0 \quad \text{Eq20} \]
\[ F_2 \cdot \sin(\alpha) \cdot L_2 \cdot \cos(\varphi_1) + F_2 \cdot \cos(\alpha) \cdot L_2 \cdot \sin(\varphi_1) - F_y \cdot L_1 \cdot \cos(\varphi_1) \]
\[ + F_x \cdot L_1 \cdot \sin(\varphi_1) = 0 \quad \text{Eq21} \]
Free Body Diagram Upper Arm

**Alternative #1:**
The rocking arms have the same geometry as the current design, where the lift links are attached outside of the upper arm and hydraulic cylinder alignment. The old spline system is replaced with a linking weldment and the arms are attached at two pivot points, UA$_{PL}$ and UA$_{PR}$. These pivot points are aligned in y-axis making the pivot points free of moment load around z- and x-axis.

![Free Body Diagram](image)

**Resulting Moments:**

- $M_{2X} = F_x \cdot \sin(\alpha) \cdot L_8$  
  \[\text{Eq27}\]
- $M_{2Z} = F_z \cdot \cos(\alpha) \cdot L_8$  
  \[\text{Eq28}\]
**Alternative #2:**
The Lift Links are attached in alignment with the Upper Arm and the Hydraulic Cylinder. The old spline system is replaced with a linking weldment and the arms are attached at the two pivot points, UA<sub>PL</sub> and UA<sub>PR</sub>. These pivot points are aligned in y-axis making the pivot points free of moment load around z- and x-axis.

The force acting upon the Upper Arm from the Lift Rod, $F_2$, can no longer be approximated in a 2D scenario.

![Figure A.10: Lift Rod aligned with the Hydraulic Cylinder](image1)

![Figure A.11: Side View of Upper Arm](image2)

![Figure A.12: Angle of $F_{2,xyz}$](image3)

**Resulting 3D Force**

$$F_{2,xyz} = \frac{F_2}{\cos(\beta)}$$

- $-R_{XL} - R_{ZR} - F_2 \cdot \sin(\alpha) + F_3 \cdot \sin(\phi_3) = 0$

- $-R_{XR} - R_{XR} + F_2 \cdot \cos(\alpha) + F_3 \cdot \cos(\phi_3) = 0$

$$R_x - F_{2,xyz} \cdot \sin(\beta) = 0$$

- $-R_{XR} \cdot L_9 + \frac{F_3}{2} \cdot \sin(\phi_3) \cdot L_9 + F_{2,xyz} \cdot \sin(\beta) \cdot L_6 \cdot \sin(\phi_4) = 0$

- $-F_3 \cdot \sin(\phi_3) \cdot L_7 \cdot \cos(\phi_4) + F_3 \cdot \cos(\phi_3) \cdot L_7 \cdot \sin(\phi_4) + F_2 \cdot \sin(\alpha) \cdot L_6 \cdot \cos(\phi_4) + F_2 \cdot \cos(\alpha) \cdot L_6 \cdot \sin(\phi_4) = 0$

- $-F_{2,xyz} \cdot \sin(\beta) \cdot L_6 \cdot \cos(\phi_4) + R_{XR} \cdot L_9 - F_3 \cdot \cos(\phi_3) \cdot L_9 = 0$
A.2.2 Load scenario of pulling an implement

The load scenario of an implement in use is mainly to analyze the stresses of the Upper Link in compression and the Lower Link in tension. The vertical force from an implement is generally downward but soil forces can be upward on the tool if for example, the soil is hard and dry or the tool cutting edges are worn [5].

The maximum load on the Upper Link in compression occurs when the implements up-force, \( F_{up,C} \), is great enough to remove weight from the rear end of the tractor \( (R_{rear}=0) \). The maximum pulling force on the lower links is equal to the tractor weight which is 100lb per horsepower. The implement is approximated to be level with the ground \( (\theta=0) \), and the implement center of gravity is estimated to be located 3 meter away from the lower hitch points \( (L_{4,C}=3) \). The tractor center of gravity is approximated to have roughly the same load ratio per wheel axle as a phase C2 with a 3pt installation (69% on the front axle). Note that the pulling scenario is only to be analyzed for the linkage lowered and should be considered a very rough approximation of an extreme scenario. Implements with the extreme pulling scenario of soil-down force is usually equipped with a tail wheel which supports this load making the vertical upward force the extreme scenario.

Free Body Diagram Implement

Figure A.13: Implement in use

<table>
<thead>
<tr>
<th>Variable description</th>
<th>Variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tractor mass, assumed 500HP engine</td>
<td>( m_{500} )</td>
<td>22,500 [kg]</td>
</tr>
<tr>
<td>Gravity</td>
<td>( g )</td>
<td>9.8 [m/s(^2)]</td>
</tr>
<tr>
<td>Ratio of wheel load</td>
<td>( R_{front} )</td>
<td>69 [%]</td>
</tr>
<tr>
<td>Ratio of wheel load</td>
<td>( R_{rear} )</td>
<td>31 [%]</td>
</tr>
<tr>
<td>Distance, front axle to Center of Gravity</td>
<td>( L_{1,C} )</td>
<td></td>
</tr>
<tr>
<td>Distance, front axle to LLP</td>
<td>( L_{2,C} )</td>
<td>4.5 [m]</td>
</tr>
<tr>
<td>Wheel base</td>
<td>( L_{3,C} )</td>
<td>4 [m]</td>
</tr>
<tr>
<td>Assumed Distance, hitch points to implement C.G</td>
<td>( L_{4,C} )</td>
<td>3 [m]</td>
</tr>
<tr>
<td>Up-force of an implement in use</td>
<td>( F_{up,C} )</td>
<td></td>
</tr>
<tr>
<td>Pulling force of an implement in use</td>
<td>( F_{pull,C} )</td>
<td></td>
</tr>
<tr>
<td>Force acting at LLHP in y (positive for mast side)</td>
<td>( F_{y,C} )</td>
<td></td>
</tr>
<tr>
<td>Force acting at LLHP in x (positive for mast side)</td>
<td>( F_{x,C} )</td>
<td></td>
</tr>
</tbody>
</table>
Force calculation for pulling scenario

| Reaction force at rear wheels | \( R_{\text{rear}, C} = 0.31 \cdot m_{500} \cdot g \) |
| Reaction force at front wheels | \( R_{\text{front}, C} = 0.69 \cdot m_{500} \cdot g \) |
| Moment around front axle (no implement attached) | \( L_1, C = \frac{L_3, C \cdot R_{\text{rear}, C}}{m_{500} \cdot g} \) |
| Up-Force of an implement at hitch points (both ULHP and LLHP) | \( F_{\text{up}, C} = \frac{m_{500} \cdot g \cdot L_1, C}{L_2, C + L_1 \cdot \cos(\phi_1)} \) |
| Pulling force | \( F_{\text{pull}, C} = m_{500} \cdot g \) |

Free Body Diagram Mast

\[
\begin{align*}
- F_{UL, C} \cdot \sin(\phi_2) + F_{y, C} + F_{\text{up}, C} &= 0 \quad \text{Eq34} \\
- F_{UL, C} \cdot \cos(\phi_2) + F_{x, C} + F_{\text{pull}, C} &= 0 \quad \text{Eq35} \\
F_{UL, C} \cdot \cos(\phi_2) \cdot L_5 + F_{\text{up}, C} \cdot L_5, C &= 0 \quad \text{Eq36}
\end{align*}
\]

Free Body Diagram Lower Link and Lift Rod

The forces are calculated for an extreme scenario where the full load is shifted to one side.

\[
\begin{align*}
L_{y, C} + F_{LR, C} \cdot \sin(\alpha) - F_{y, C} &= 0 \quad \text{Eq19} \\
L_{x, C} - F_{LR, C} \cdot \cos(\alpha) - F_{x, C} &= 0 \quad \text{Eq20} \\
F_{LR, C} \cdot \sin(\alpha) \cdot L_2 \cdot \cos(\phi_1) + F_{LR, C} \cdot \cos(\alpha) \cdot L_2 \cdot \sin(\phi_1) - F_{y, C} \cdot L_1 \cdot \cos(\phi_1) - F_x \cdot L_1 \cdot \sin(\phi_1) &= 0 \quad \text{Eq21}
\end{align*}
\]
A.3 Stresses

The factor $K_t$ is applied for approximating the increased stresses around the hole for a plate in tension (or compression). Variable $d$ is the pin diameter and $w$ the surrounding material width.

$$K_t = 3.0039 - 3.753 \left( \frac{d}{w} \right) + 7.973 \left( \frac{d}{w} \right)^2 - 9.2659 \left( \frac{d}{w} \right)^3 + 1.814 \left( \frac{d}{w} \right)^5$$

(Eq34 (Norton 2000))

A.3.1 Lift Rod

For the current version, the Lift Rod is attached to Lower Link using a clevis structure and to the upper arm using a ball joint. If the option “alternative #2” is used for the upper arm, both the upper and lower attachment is to have a clevis structure. All pins are analyzed for equally distributed double sheer.

For maximum load is calculated for lifting scenario in accordance with section A.2.1. For analysis of pulling scenario, force $F_{2,xyz}$ is swapped with variable $F_{LR,c}$ (stated in section A.2.2).

---

**Sheer on Pins**

<table>
<thead>
<tr>
<th>Sheer force (double shear)</th>
<th>$T_{LR,Pin} = \frac{F_{2,xyz}}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin Cross-Section</td>
<td>$A_{LR,Pin} = \pi \cdot d_{LR,Pin}^2$</td>
</tr>
<tr>
<td>Shear stress average</td>
<td>$\tau_{LR,Pin} = \frac{T_{LR,Pin}}{A_{LR,Pin}}$</td>
</tr>
<tr>
<td>Jouravski-Factor for Circular Cross-Section $\mu=2.0$</td>
<td>$\tau_{LR,Pin,\max} = \mu \cdot \tau_{LR,Pin}$</td>
</tr>
</tbody>
</table>

**Stresses at critical points of Clevis Attachment**

- Stress in clevis material around pin, $K_t$ is in accordance with Eq34 for $d/w$ equals $d_{LR,pin}/(d_{LR,pin} + 2r_{LR,cl})$
  - $A_{LR,cl} = \frac{t_{LR,cl}}{r_{LR,cl}^{4}}$
  - $\sigma_{LR,cl} = \frac{F_{2,xyz}}{A_{LR,cl}}$
  - $\sigma_{LR,cl,\max} = K_t \cdot \sigma_{LR,cl}$

- Stress in clevis material at pin, the cross-section area is approximated to the pin diameter times the material width
  - $A_{LR,cl,Pin} = \frac{d_{LR,Pin}}{t_{LR,cl}^{2}}$
  - $\sigma_{LR,cl,Pin} = \frac{F_{2,xyz}}{A_{LR,cl,Pin}}$
For tearout failure, the cross-section area is evaluated as follows (Norton 2000)

\[ A_{LR, \text{ch, tearout}} = r_{LR, \text{ch}} t_{LR, \text{ch}}^2 \]

\[ \sigma_{LR, \text{ch, tearout}} = \frac{F_{2,xyz}}{A_{LR, \text{ch, tearout}}} \]

**Stresses in threaded Rod**

Tension in threaded rod. The cross-section area of the threaded rod is approximated to take into account stress concentrations. \( p_{LR} \) equals thread pitch. (Norton 2000)

\[ d_{p,LR} = d_{LR} - 0.649519 \cdot p_{LR} \]

\[ d_{r,LR} = d_{LR} - 1.226869 \cdot p_{LR} \]

\[ A_{LR, \text{rod}} = \frac{\pi}{4} \left( \frac{d_{p,LR} + d_{r,LR}}{2} \right)^2 \]

\[ \sigma_{LR} = \frac{F_{2,xyz}}{A_{LR, \text{rod}}} \]

**Stresses in threads of the rod (per thread)**

Rod evaluated for inner diameter of threads, \( d_{LR, i} \), \( w_{i,LR} = 0.8 \) for ISO thread (inner) (Norton 2000)

\[ A_{LR, \text{thread}} = \pi \cdot d_{LR, i} \cdot w_{i,LR} \cdot p_{LR} \]

\[ \tau_{LR, \text{thread}} = \frac{F_{2,xyz}}{A_{LR, \text{thread}}} \]

**Stresses in threads of clevis and rod link (the nut) (per thread)**

Nut evaluated for outer diameter of threads, \( d_{LR, o} \), \( w_{o,LR} = 0.88 \) for ISO thread (outer) (Norton 2000)

\[ A_{LR, \text{nut, thread}} = \pi \cdot d_{LR, o} \cdot w_{o,LR} \cdot p_{LR} \]

\[ \tau_{LR, \text{nut, thread}} = \frac{F_{2,xyz}}{A_{LR, \text{nut, thread}}} \]

**Stress at rod link, (connecting the two threaded rods)**

\[ A_{LR, RL} = \frac{(d_{LR, RL}^2 - d_{LR}^2) \cdot \pi}{4} \]

\[ \sigma_{LR, RL} = \frac{F_{2,xyz}}{A_{LR, RL}} \]

**Stress at pin for locking free float**

Area of free float pin

\[ A_{LR, \text{ffpin, C}} = \frac{\pi \cdot d_{LR, \text{ffpin}}^2}{4} \]

Average sheer on free float pin (note only for pulling scenario)

\[ \tau_{LR, \text{ffpin, C}} = \frac{T_{LR, \text{Pin, C}}}{A_{LR, \text{ffpin, C}}} \]

Jouravski-Factor for Circular Cross-Section \( \mu = 2.0 \)

\[ \tau_{LR, \text{ffpin, max, C}} = \mu \cdot \tau_{LR, \text{ffpin, C}} \]
A.3.2 Upper Link

The Upper Link is evaluated for both ball joint and clevis joints. For the ball joint, gussets are added to the transition from threaded rod to ball for extra support. Also considered is a clevis attachment where the clevis end connects to the threaded rod. Maximum stresses occur for pulling scenario in accordance with section A.2.2. Therefore, analysis with $F_1$ swapped with $F_{UL, \text{Com}}$ Compression acts as the decisive case scenario.

![Figure A.15: Upper Link clevis cross section at pivot points](image)

**Sheer on Pins**

<table>
<thead>
<tr>
<th>Sheer force (double shear)</th>
<th>$T_{UL, Pin} = \frac{F_1}{2}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Pin Cross-Section</th>
<th>$A_{UL, Pin} = \frac{\pi \cdot d_{UL, Pin}^2}{4}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Shear stress average</th>
<th>$\tau_{UL, Pin} = \frac{T_{UL, Pin}}{A_{UL, Pin}}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Jouravski-Factor for Circular Cross-Section $\mu=2.0$</th>
<th>$\tau_{UL, Pin, max} = \mu \cdot \tau_{UL, Pin}$</th>
</tr>
</thead>
</table>

**Stresses at critical points of Clevis Attachment**

<table>
<thead>
<tr>
<th>Stress in material around the pin at the attachment. $K_i$ is in accordance with Eq34 for $d/w$ equals $d_{ul, pin}/(d_{ul, pin}+2r_{ul, ma})$ Note: calculated as an unsupported hole, a rough approximation</th>
<th>$A_{UL, ma} = \frac{t_{UL, ma} \cdot d_{UL, ma} \cdot 2}{F_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{UL, ma}$</td>
<td>$A_{UL, ma}$</td>
</tr>
<tr>
<td>$\sigma_{UL, ma, max} = K_i \cdot \sigma_{UL, ma}$</td>
<td>$\sigma_{UL, ma}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stress in clevis material at pin, the cross-section area is approximated to the pin diameter times the material width</th>
<th>$A_{UL, ma, pin} = \frac{d_{UL, pin} \cdot t_{UL, ma}}{F_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{UL, ma, pin}$</td>
<td>$\sigma_{UL, ma, pin}$</td>
</tr>
</tbody>
</table>

For tearout failure, the cross-section area is evaluated as follows (Norton 2000)

<table>
<thead>
<tr>
<th>$A_{UL, ma, tearout}$</th>
<th>$\frac{r_{UL, ma} \cdot 4F_1}{t_{UL, ma}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{UL, ma, tearout}$</td>
<td>$A_{UL, ma, tearout}$</td>
</tr>
</tbody>
</table>
### Stresses in threaded Rod

Tension in threaded rod. The cross-section area of the threaded rod is approximated to take account of stress concentrations. $p_{UL}$ equals thread pitch. (Norton 2000)

\[
d_{p, UL} = d_{UL} - 0.649519 p_{UL} \\
d_{r, UL} = d_{UL} - 1.226869 p_{UL} \\
A_{UL, rod} = \frac{\pi}{4} \left( \frac{d_{p, UL} + d_{r, UL}}{2} \right)^2 \\
\sigma_{UL} = \frac{F_1}{A_{UL, rod}}
\]

#### Stresses in threads of the rod (per thread)

Rod evaluated for inner diameter of threads, $d_{LR}$, $w_{i,LR} = 0.8$ for ISO thread (inner) (Norton 2000)

\[
A_{LR, thread} = \pi \cdot d_{LR} \cdot i \cdot w_{i,LR} \cdot P_{LR} \\
\tau_{UL, thread} = \frac{F_1}{A_{UL, thread}}
\]

#### Stresses in threads of clevis and rod link (the nut) (per thread)

Nut evaluated for outer diameter of threads, $d_{LRo}$, $w_{i,LR} = 0.88$ for ISO thread (outer) (Norton 2000)

\[
A_{UL, nut, thread} = \pi \cdot d_{UL} \cdot w_{o,UL} \cdot P_{UL} \\
\tau_{UL, nut, thread} = \frac{F_1}{A_{UL, nut, thread}}
\]

#### Stress at rod link connecting the two threaded rods (Adjustment Tube)

Stress in connecting tube link

\[
A_{UL, RL} = \left( \frac{d_{UL, RL}^2 - d_{UL}^2}{4} \right) \cdot \pi \\
\sigma_{UL, RL} = \frac{F_1}{A_{UL, RL}}
\]
The Upper Link Attachment (ULP) for a PTO installation is analyzed for the pulling scenario from section A.2.2. The Upper Link is loaded in compression where the negative force $F_{UL,C}$ is preset as $F_{UL,compression}$ positive. The structure is analyzed for what is assumed critical points A, B and C.

### Measurements

**Distance from exerted forces to point A,B,C and D on x-axis.** (roughly)

$$L_{5,ULP} = \frac{L_{2,ULP}}{2} - \frac{L_{1,ULP}}{2}$$

**$e_Z$ – distance to C.G.**

**Area**

$$A_{i,ULP} = a_{i,ULP} \cdot b_{i,ULP} \quad \text{for } i = 1 \text{ to } 4$$

$$A_{TOT,ULP} = \sum_{i=1}^{4} a_{i,ULP} \cdot b_{i,ULP}$$

**Distance from plate i to C.G.**

$$(2 \text{ and } 4 \text{ are to have the same geometry})$$

$$e_{1,ULP} = \frac{b_{1,ULP}}{2}$$

$$e_{2,ULP} = b_{1,ULP} + \frac{a_{2,ULP}}{2}$$

$$e_{3,ULP} = b_{1,ULP} + \frac{a_{3,ULP}}{2}$$

$$e_{4,ULP} = -e_{2,ULP}$$

$$e_{Z,ULP} = \frac{\sum_{i=1}^{4} e_{i,ULP} \cdot A_{i,ULP}}{A_{TOT,ULP}}$$

**Distance to critical points**

$$z_{A,ULP} = -e_{Z,ULP} = z_{D,ULP}$$

$$z_{B,ULP} = b_{1,ULP} + a_{2,ULP} - e_{Z,ULP}$$

$$z_{C,ULP} = b_{1,ULP} + a_{3,ULP} - e_{Z,ULP}$$

$$y_{A,ULP} = a_{1,ULP} \cdot \frac{1}{2}$$

$$y_{B,ULP} = -a_{1,ULP} \cdot \frac{1}{2}$$

$$y_{C,ULP} = 0$$

$$y_{D,ULP} = b_{1,ULP} \cdot \frac{1}{2}$$

---

Figure A.16: ULP with PTO loaded in compression
### Reaction Loadings at Side Plate

#### Forces

\[
F_{z, ULP} = \frac{F_{UL, compression}}{2} \cdot \cos(-\varphi_2)
\]
\[
F_{y, ULP} = \frac{F_{UL, compression}}{2} \cdot \sin(-\varphi_2)
\]

#### Moments

\[
M_{x, ULP} = F_{z, ULP} \cdot L_3, ULP
\]
\[
M_{y, ULP} = -F_{z, ULP} \cdot L_5, ULP
\]
\[
M_{z, ULP} = -F_{y, ULP} \cdot L_5, ULP
\]

#### Moment of Inertia

Plate 2 and 4 have the same contribution

\[
l_y, ULP = \left( \frac{a_{1, ULP} \cdot b_{1, ULP}^3}{12} + \left( e_{1, ULP} - e_{z, ULP} \right)^2 \cdot A_{1, ULP} \right)
\]
\[
+ \left( \frac{b_{2, ULP} \cdot a_{2, ULP}^3}{12} + \left( e_{2, ULP} - e_{z, ULP} \right)^2 \cdot A_{2, ULP} \right)^2
\]
\[
+ \left( \frac{b_{3, ULP} \cdot a_{3, ULP}^3}{12} + \left( e_{3, ULP} - e_{z, ULP} \right)^2 \cdot A_{3, ULP} \right)
\]
\[
l_z, ULP = \frac{b_{1, ULP} \cdot a_{1, ULP}^3}{12} + \left( \frac{a_{2, ULP} \cdot b_{2, ULP}^3}{12} + \left( \frac{a_{1, ULP}}{2} - \frac{b_{2, ULP}}{2} \right)^2 \cdot A_{2, ULP} \right)
\]
\[
- \left( \frac{b_{2, ULP}}{2} \right)^2 \cdot A_{2, ULP}
\]
\[
\frac{a_{3, ULP} \cdot b_{3, ULP}^3}{12}
\]

#### Stresses at Critical Points

\[
\sigma_{xx, ULP,j} = \frac{M_{y, ULP} \cdot y_{j, ULP}}{l_y, ULP} - \frac{M_{z, ULP} \cdot y_{j, ULP}}{l_z, ULP} \quad \text{for } j = A..D
\]

#### Shear Contribution

#### Torsional Stiffness of the Plates

\[
K_{v,i, ULP} = \frac{1}{3} \cdot a_{i, ULP} \cdot b_{i, ULP}^3 \quad \text{for } i = 1..4
\]

#### Section Modules in Torsion

\[
W_{v,i, ULP} = \frac{1}{3} \cdot a_{i, ULP} \cdot b_{i, ULP}^2 \quad \text{for } i = 1..4
\]

First moment of cross-sectional area \( Q \) for point C and D, and thickness \( t \) of material at C and D.

\[
Q_{C, z, ULP} = \left( \frac{b_{3, ULP} \cdot a_{3, ULP}}{2} \right) \cdot \frac{b_{3, ULP}}{4} + \left( a_{1, ULP} \cdot b_{1, ULP} \right) \cdot \frac{a_{1, ULP}}{2}
\]
\[
+ \left( a_{2, ULP} \cdot b_{2, ULP} \right) \cdot \left( \frac{a_{1, ULP}}{2} - \frac{b_{2, ULP}}{2} \right)
\]
\[
t_C, ULP = b_{1, ULP} + a_{3, ULP}
\]
\[
Q_{D, z, ULP} = \left( \frac{a_{1, ULP} \cdot b_{1, ULP}}{2} \right) \cdot \frac{a_{1, ULP}}{2}
\]
\[
+ \left( a_{2, ULP} \cdot b_{2, ULP} \right) \cdot \left( \frac{a_{1, ULP}}{2} - \frac{b_{2, ULP}}{2} \right)
\]
\[
t_D, ULP = b_{1, ULP}
\]
Stress expressions and principles:

Shear stress at critical point for torsion calculus of joint members (Dahlberg 2001):

\[
\tau_{\text{max}, A, \text{ULP}} = \frac{M_x, \text{ULP}}{W_v, 1, \text{ULP}} \left( K_{v, 1, \text{ULP}} \sum_{i=1}^{4} K_{v, i, \text{ULP}} + \frac{F_{v, \text{ULP}} Q_{1, z, \text{ULP}}}{I_p^{\text{ULP}} I_D, \text{ULP}} \right)
\]

\[
\tau_{\text{max}, D, \text{ULP}} = \frac{M_x, \text{ULP}}{W_v, 1, \text{ULP}} \left( K_{v, 1, \text{ULP}} \sum_{i=1}^{4} K_{v, i, \text{ULP}} + \frac{F_{v, \text{ULP}} Q_{1, z, \text{ULP}}}{I_p^{\text{ULP}} I_D, \text{ULP}} \right)
\]

\[
\tau_{\text{max}, B, \text{ULP}} = \frac{M_x, \text{ULP}}{W_v, 2, \text{ULP}} \left( K_{v, 2, \text{ULP}} \sum_{i=1}^{4} K_{v, i, \text{ULP}} \right)
\]

\[
\tau_{\text{max}, C, \text{ULP}} = \frac{M_x, \text{ULP}}{W_v, 3, \text{ULP}} \left( K_{v, 3, \text{ULP}} \sum_{i=1}^{4} K_{v, i, \text{ULP}} + \frac{F_{v, \text{ULP}} Q_{1, z, \text{ULP}}}{I_p^{\text{ULP}} I_D, \text{ULP}} \right)
\]

**Stresses**

**Principle stresses**

\[
\sigma_{1, \text{ULP}, j} = \left( \frac{\sigma_{xx, \text{ULP}, j}}{2} \right) + \sqrt{\left( \left( \frac{\sigma_{xx, \text{ULP}, j}}{2} \right)^2 + \tau_{\text{max}, j, \text{ULP}}^2 \right)}
\]

\[
\sigma_{2, \text{ULP}, j} = \left( \frac{\sigma_{xx, \text{ULP}, j}}{2} \right) - \sqrt{\left( \left( \frac{\sigma_{xx, \text{ULP}, j}}{2} \right)^2 + \tau_{\text{max}, j, \text{ULP}}^2 \right)}
\]

**Equivalent stress**

\[
\sigma_{\text{eff, UL P, j}} = \text{evalf} \left( \sigma_{1, \text{ULP}, j} - \sigma_{2, \text{ULP}, j} \right) \text{ for } j = A..D
\]
### A.3.3 Hydraulic Cylinders

The Hydraulic Cylinders attached to the drawbar cage with a solid plate structure, figure A.17, or at the upper arm with a clevis structure, figure A.18.

![Figure A.17: Hydraulic Cylinder, clevis attach](image1)

![Figure A.18: Hydraulic Cylinder solid plate attachment](image2)

<table>
<thead>
<tr>
<th>Sheer on pivoting pins</th>
<th>$T_{HC, Pin} = \frac{1}{2} \cdot \frac{F_3}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-section pin</td>
<td>$A_{HC, Pin} = \frac{\pi \cdot d_{HC, Pin}^2}{4}$</td>
</tr>
<tr>
<td>Shear stress average</td>
<td>$\tau_{HC, Pin} = \frac{T_{HC, Pin}}{A_{HC, Pin}}$</td>
</tr>
<tr>
<td>Jouravski-Factor for Circular Cross-Section $\mu = 2.0$</td>
<td>$\tau_{HC, Pin, max} = \mu \cdot \tau_{HC, Pin}$</td>
</tr>
</tbody>
</table>

#### Stresses analysis for varying failures

<table>
<thead>
<tr>
<th>Stress in material at pin, cross-section area is approximated to the pin diameter times the width</th>
<th>$A_{HC, ma, pin} = d_{HC, Pin} \cdot t_{HC, ma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{HC, ma, pin} = \frac{F_3}{2 \cdot A_{HC, ma, pin}}$</td>
<td></td>
</tr>
<tr>
<td>Stress in material at solid attachment analyzed for tearout failure. Tearout cross-section area is estimated as follows (Norton 2000)</td>
<td>$A_{HC, ma, tearout} = r_{HC, ma} \cdot t_{HC, ma}$</td>
</tr>
<tr>
<td>$\sigma_{HC, ma, tearout} = \frac{F_3}{2 \cdot A_{HC, ma, tearout}}$</td>
<td></td>
</tr>
<tr>
<td>Stress in material at clevis pin. Cross-section area is approximated to the pin diameter times the width</td>
<td>$A_{HC, cl, pin} = d_{HC, Pin} \cdot t_{HC, cl} \cdot t_{HC, pin}$</td>
</tr>
<tr>
<td>$\sigma_{HC, cl, pin} = \frac{F_3}{2 \cdot A_{HC, cl, pin}}$</td>
<td></td>
</tr>
</tbody>
</table>

#### Pressure in Hydraulic Cylinders

<table>
<thead>
<tr>
<th>Cross section area of piston in hydraulic cylinder</th>
<th>$A_{HC} = \frac{\pi \cdot d_{HC}^2}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic Cylinder Fluid Pressure</td>
<td>$\sigma_{HC} = \frac{F_3}{2 \cdot A_{HC}}$</td>
</tr>
</tbody>
</table>
### Upper Clevis

**Figure A. 19: Clevis Geometry**

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to A, x-Axis</td>
<td>$L_{A, HC, cl} = \frac{L_{1, HC, cl}}{2} + \frac{L_{2, HC, cl}}{2} - \frac{L_{3, HC, cl}}{2}$</td>
</tr>
<tr>
<td><strong>Moment of Inertia</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{y, HC, cl} = \frac{b_{HC, cl} h_{HC, cl}^3}{12}$</td>
</tr>
<tr>
<td>Distance to A, z-Axis</td>
<td>$z_{A, HC, cl} = \frac{h_{HC, cl}}{2}$</td>
</tr>
<tr>
<td><strong>Moment around y-axis</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$M_{y, HC, cl} = -\frac{F_{max, HC, cl}}{2} \cdot L_{A, HC, cl}$</td>
</tr>
<tr>
<td><strong>Stress at A</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{A, HC, cl} = \frac{M_{y, HC, cl} z_{A, HC, cl}}{I_{y, HC, cl}}$</td>
</tr>
</tbody>
</table>
A.3.4  Lower Link

The pin at the hitch point is designed to meet with standards for implements and therefore differs from the attachment pin to the rear frame. The material stress around the Lift Rod Pin is approximated with a circular cross-section around the pin (note ball geometry neglected).

Resulting force magnitude at pivoting points

<table>
<thead>
<tr>
<th>Resulting Force at LL&lt;sub&gt;P&lt;/sub&gt;</th>
<th>$L.L.L = \sqrt{\left(\frac{F_x^2}{r_x^2} + \frac{F_y^2}{r_y^2}\right)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resulting Force at LL&lt;sub&gt;HP&lt;/sub&gt;</td>
<td>$L.L.F = \sqrt{\left(\frac{F_x^2}{r_x^2} + \frac{F_y^2}{r_y^2}\right)}$</td>
</tr>
</tbody>
</table>

Sheer on pin at LLP

<table>
<thead>
<tr>
<th>Sheer on pin at LL&lt;sub&gt;P&lt;/sub&gt; (double sheer)</th>
<th>$T_{LLP,Pin} = \frac{L.L.L}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ LLP,Pin } = \frac{\pi \cdot d_{ LLP,Pin }^2}{4}$</td>
<td>$\tau_{ LLP,Pin } = \frac{T_{ LLP,Pin } \cdot d_{ LLP,Pin } \cdot \mu}{A_{ LLP,Pin } \cdot \mu}$</td>
</tr>
</tbody>
</table>

Jouravski-Factor for Circular Cross-Section $\mu=2.0$

Stresses analysis for varying failures at LLP

<table>
<thead>
<tr>
<th>Stress in LLP material at pin. Cross-section area is approximated to the pin diameter times the width</th>
<th>$A_{ LLP,Pin } = \frac{d_{ LLP,Pin } \cdot t_{ LLP,Pin }}{A_{ LLP,Pin } \cdot \mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress at LLP analyzed for tearout failure. Cross section area is evaluated as follows: (Norton 2000)</td>
<td>$A_{ LLP,tearout } = \frac{t_{ LLP,ma } \cdot t_{ LLP,ma }^2}{A_{ LLP,tearout } \cdot \mu}$</td>
</tr>
<tr>
<td>Stress in material around the pin at LLP attachment. $K_i$ in accordance with Eq34 for $d/w$ equals $d_{ LLP,Pin }/(d_{ LLP })$</td>
<td>$A_{ LLP,ma } = \frac{t_{ LLP,ma } \cdot t_{ LLP,ma }^2}{A_{ LLP,ma } \cdot \mu}$</td>
</tr>
<tr>
<td>Stress: calculated as unloaded hole, a very rough approximation</td>
<td>$\sigma_{ LLP,ma } = \frac{L.L.L}{A_{ LLP,ma } \cdot \mu}$</td>
</tr>
<tr>
<td>Note: Ball geometry is neglected</td>
<td>$\sigma_{ LLP,ma, max } = K_i \cdot \sigma_{ LLP,ma }$</td>
</tr>
</tbody>
</table>
Stresses analysis for varying failures at LLLR

Stress in material at lift rod pin, cross-section area is approximated to the pin diameter times the width

\[ A_{\text{LLLR}, \text{pin}} = d_{\text{LR, bottom, Pin}} \cdot t_{\text{LL}, LR} \]

\[ \sigma_{\text{LLLR}, \text{pin}} = \frac{F_{2,xyz}}{A_{\text{LLLR}, \text{pin}}} \]

Stress in material analyzed for tearout failure, cross-section area is evaluated as follows:

\[ A_{\text{LLLR}, \text{tearout}} = \frac{d_{\text{LR, LR, bottom, Pin}} - d_{t, LR} - \frac{1}{2}}{t_{\text{LL}, LR}} \]

\[ \sigma_{\text{LLLR, tearout}} = \frac{F_{2,xyz}}{A_{\text{LLLR}, \text{tearout}}} \]

Forces converted to coordinate system fixed to the link.

\[ \begin{bmatrix} L_L \\ -L_H \end{bmatrix} = \begin{bmatrix} I_x \\ -I_y \end{bmatrix} \]

\[ L_{\text{HP}} = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} \]

\[ F_{\text{LLLR}} = \begin{bmatrix} -F_{2,xyz} \cos(\beta) \cos(\alpha) \\ F_{2,xyz} \cos(\beta) \sin(\alpha) \end{bmatrix} \]

\[ \chi_{\text{LL}} = -\varphi_1 \]

\[ M_{\text{rot, LL}} = \begin{bmatrix} \cos(\chi_{\text{LL}}) & -\sin(\chi_{\text{LL}}) \\ \sin(\chi_{\text{LL}}) & \cos(\chi_{\text{LL}}) \end{bmatrix} \]

\[ L_{\text{P, rot}} = M_{\text{rot, LL}} \cdot L_L \]

\[ L_{\text{HP, rot}} = M_{\text{rot, LL}} \cdot L_{\text{HP}} \]

\[ F_{\text{LLLR, rot}} = M_{\text{rot, LL}} \cdot F_{\text{LLLR}} \]

Stresses in cross section at lift rod attachment

\[ A_{i, \text{LL}, i} = a_{i, \text{LL}, i} \cdot b_{i, \text{LL}, i} \text{ for } i = 1, 2, 3 \]

\[ A_{\text{rot, LL}, i} = 2 \cdot A_{i, \text{LL}, i} + A_{2, \text{LL}, i} + A_{3, \text{LL}, i} \]

Distance to C.G.

\[ e_{1, \text{LL}, i} = \frac{a_{2, \text{LL}, i} + b_{1, \text{LL}, i}}{2} \]

\[ e_{3, \text{LL}, i} = \frac{a_{2, \text{LL}, i}}{2} + b_{1, \text{LL}, i} + \frac{b_{3, \text{LL}, i}}{2} \]

Moment of Inertia

\[ I_{2, \text{LL}, i} = \frac{a_{1, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + a_{2, \text{LL}, i}^2 + \frac{a_{3, \text{LL}, i}^2}{12} \]

\[ I_{3, \text{LL}, i} = \frac{a_{1, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{2, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{3, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} \]

\[ I_{4, \text{LL}, i} = \frac{a_{1, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{2, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{3, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} \]

\[ I_{5, \text{LL}, i} = \frac{a_{1, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{2, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} + \frac{a_{3, \text{LL}, i} b_{1, \text{LL}, i}^2}{12} \]

Bending Moment at LLLR (C for pulling)

\[ M_{\text{LLLR}} = -L_{\text{HP, rot}} \cdot y \cdot (L_1 - L_2) \]

\[ M_{\text{LLLR, C}} = -L_{\text{HP, rot, C, y}} \cdot (L_1 - L_2) \]

Casting with I-beam geometry
<table>
<thead>
<tr>
<th>Distance to critical point A and B</th>
<th>$z_{A, LL, I} = \frac{a_{2, LL, I}}{2} - b_{1, LL, I} - \frac{b_{3, LL, I}}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{B, LL} = \frac{a_{2, LL, I}}{2} + b_{1, LL} + \frac{b_{3, LL, I}}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stresses at lifting</th>
<th>$\sigma_{xx, A, I} = \frac{ML_{c, x}}{A_{tot, LL, I}} + \frac{M_{LLRR}z_{A, LL, I}}{I_{y, LL, I}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx, B, I} = \frac{ML_{c, x}}{A_{tot, LL, I}} + \frac{M_{LLRR}z_{B, LL, I}}{I_{y, LL, I}}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stresses at pulling (implement lowered)</th>
<th>$\sigma_{xx, A, I, C} = \frac{ML_{c, x}}{A_{tot, LL, I}} + \frac{M_{LLRR}z_{A, LL, I}}{I_{y, LL, I}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx, B, I, C} = \frac{ML_{c, x}}{A_{tot, LL, I}} + \frac{M_{LLRR}z_{B, LL, I}}{I_{y, LL, I}}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bar geometry</th>
<th>$A_{LL, bar} = a_{LL, bar} \cdot b_{LL, bar} = a_{LL, bar} \cdot b_{LL, hole}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Cross section area</th>
<th>$A_{LL, bar} = a_{LL, bar} \cdot b_{LL, bar} = a_{LL, bar} \cdot b_{LL, hole}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Moment of Inertia</th>
<th>$I_{y, LL, bar} = \frac{a_{LL, bar} \cdot b_{LL, bar}^3}{12} - \frac{a_{LL, bar} \cdot b_{LL, hole}^3}{12}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Distance to critical point A and B</th>
<th>$z_{A, LL, bar} = \frac{b_{LL, bar}}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{B, LL, bar} = \frac{b_{LL, bar}}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stresses at lifting</th>
<th>$\sigma_{xx, A, bar} = \frac{ML_{c, x}}{A_{LL, bar}} + \frac{M_{LLRR}z_{A, LL, bar}}{I_{y, LL, bar}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx, B, bar} = \frac{ML_{c, x}}{A_{LL, bar}} + \frac{M_{LLRR}z_{B, LL, bar}}{I_{y, LL, bar}}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stresses at pulling</th>
<th>$\sigma_{xx, A, bar, C} = \frac{ML_{c, x}}{A_{LL, bar}} + \frac{M_{LLRR}z_{A, LL, bar}}{I_{y, LL, bar}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx, B, bar, C} = \frac{ML_{c, x}}{A_{LL, bar}} + \frac{M_{LLRR}z_{B, LL, bar}}{I_{y, LL, bar}}$</td>
<td></td>
</tr>
</tbody>
</table>
## A.3.5 Upper Arm at Attachment Points

The Upper arm is analyzed at its pivot points. The material stress around the pin is approximated with a circular cross-section.

![Figure A.24: Solid Plate at the Upper Arm Attachment](image)

<table>
<thead>
<tr>
<th>Resulting Force at the right side, $U_{A_P,R}$</th>
<th>$R_L = \sqrt{\left(R_{XL}^2 + R_{ZL}^2\right)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resulting Force at the left side, $U_{A_P,L}$</td>
<td>$R_R = \sqrt{\left(R_{XR}^2 + R_{ZR}^2\right)}$</td>
</tr>
</tbody>
</table>

Sheer on pins at $U_{A_P,L}$ and $U_{A_P,R}$ (left/right)

<table>
<thead>
<tr>
<th>Shear on pins at $U_{A_P,L}$ and $U_{A_P,R}$ (left/right)</th>
<th>$\tau_{UAP,L, Pin} = \frac{R_L}{2}$</th>
<th>$\tau_{UAP,R, Pin} = \frac{R_R}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{UAP, Pin}$ = $\frac{\pi c_{UAP, Pin}^2}{4}$</td>
<td>$A_{UAP, Pin}$ = $\frac{T_{UAP, Pin}}{A_{UAP, Pin}}$</td>
<td></td>
</tr>
<tr>
<td>$\tau_{UAP,L, Pin}$ = $\frac{T_{UAP,L, Pin}}{A_{UAP, Pin}}$</td>
<td>$\tau_{UAP,R, Pin}$ = $\frac{T_{UAP,R, Pin}}{A_{UAP, Pin}}$</td>
<td></td>
</tr>
</tbody>
</table>

Jouravski-Factor for Circular Cross-Section $\mu=2.0$

| Jouravski-Factor for Circular Cross-Section $\mu=2.0$ | $\tau_{UAP,L, Pin, max} = \mu \cdot \tau_{UAP,L, Pin}$ | $\tau_{UAP,R, Pin, max} = \mu \cdot \tau_{UAP,R, Pin}$ |

Stress at UAP pin

| Stress at UAP pin | $A_{UAP, pin} = \frac{d_{UAP, Pin}}{A_{UAP, pin}}$ | $\sigma_{UAP, pin} = \frac{R_R}{A_{UAP, pin}}$ |

Tearout failure at UAP

| Tearout failure at UAP | $A_{UAP, tearout} = \frac{\tau_{UAP, ma}}{A_{UAP, tearout}}$ | $\sigma_{UAP, tearout} = \frac{R_R}{A_{UAP, tearout}}$ |

Stress at UAHC pin

| Stress at UAHC pin | $A_{UAHC, pin} = \frac{d_{UAHC, Pin}}{A_{UAHC, pin}}$ | $\sigma_{UAHC, pin} = \frac{F_{3, max}}{2 \cdot A_{UAHC, pin}}$ |

Stress at UALR pin

| Stress at UALR pin | $A_{UALR, pin} = \frac{d_{UALR, Pin}}{A_{UALR, pin}}$ | $\sigma_{UALR, pin} = \frac{F_{2,xyz}}{A_{UALR, pin}}$ |

Tearout failure at UALR

| Tearout failure at UALR | $A_{UALR, tearout} = \frac{\tau_{UALR, ma}}{A_{UALR, tearout}}$ | $\sigma_{UALR, tearout} = \frac{F_{2,xyz}}{A_{UALR, tearout}}$ |
A.3.6 Upper Arm Structure

The upper arm structure is analyzed for bending at the cross section where the tube is welded and the cross section where the hydraulic cylinder attaches. The stresses are approximated from a scenario where the cross-section at this zy-plane is considered fixed. Note that the upper arm is analyzed using beam theory. Beam theory applies when den length L is much greater than the height h (L>>h). The values are therefore considered as rough estimations. All the structures are analyzed for the scenario with the Upper Arm angle $\varphi_4 = 0$.

Figure A.25: Upper arm, side view

Figure A.26: Upper arm at tube, side view
Cross-Section H

The cross-section of an H-beam with a supporting plate at the top is analyzed for alternative #2 where the hydraulic cylinder and the lift rod are aligned.

Axial Load

\[ N_{UA} = F_2 \cdot \cos(\alpha) + \frac{F_3}{2} \cdot \cos(\phi_3) \]

Moment

\[ M_{z,UA} = F_{2,yz} \cdot \sin(\beta) \cdot (L_6 - L_{10}) \]
\[ M_{y,UA} = \frac{F_3}{2} \cdot \sin(\phi_3) \cdot (L_7 - L_{10}) - F_2 \cdot \sin(\alpha) \cdot (L_6 - L_{10}) \]

Area Total

\[ A_{i,UA,H} = a_{i,UA,H} \cdot b_{i,UA,H} \quad \text{for} \quad i = 1, A \]

\[ A_{tot,UA,H} = \sum_{i = 1}^{A} A_{i,UA,H} \]

Centroid distance

\[ e_{1,H} = e_{3,H} - a_{4,UA,H} + \frac{b_{1,UA,H}}{2} \]
\[ e_{2,H} = 2 \cdot e_{1,UA,H} - 0.05 \]
\[ e_{4,H} = \frac{a_{4,UA,H}}{2} \]

\[ e_{z,H} = \frac{\sum_{i = 1}^{A} e_{i,H} \cdot A_{i,UA,H}}{A_{tot,UA,H}} \]

Point A

\[ z_{A,UA,H} = -e_{z,H} \]
\[ y_{A,UA,H} = -\frac{b_{2,UA,H}}{2} - \frac{a_{1,UA,H}}{2} \]

Moment of Inertia

\[ I_{y,UA,H} = 2 \left[ \frac{(a_{1,UA,H} \cdot b_{1,UA,H})^3}{12} + (e_{z,H} - e_{1,H})^2 \cdot A_{1,UA,H} \right] \]
\[ + \left[ \frac{(b_{2,UA,H} \cdot a_{2,UA,H})^3}{12} + (e_{z,H} - e_{2,H})^2 \cdot A_{2,UA,H} \right] \]
\[ + \left[ \frac{(b_{4,UA,H} \cdot a_{3,UA,H})^3}{12} + (e_{z,H} - e_{4,H})^2 \cdot A_{4,UA,H} \right] \]
\[ I_{z,UA,H} = \left[ \frac{(b_{1,UA,H} \cdot a_{1,UA,H})^3}{12} + (e_{z,H} - e_{1,H})^2 \cdot A_{1,UA,H} \right] \cdot 2 \]
\[ + \frac{a_{2,UA,H} \cdot b_{2,UA,H}^3}{12} + \frac{a_{4,UA,H} \cdot b_{4,UA,H}^3}{12} \]

Stress at A

\[ \sigma_{xx,UA,H} = \frac{N_{UA}}{A_{tot,UA,H}} + \frac{M_{H,UA} \cdot z_{A,UA,H}}{I_{y,UA,H}} - \frac{M_{z,UA} \cdot y_{A,UA,H}}{I_{z,UA,H}} \]
Cross-Section PL (solid plate)

The cross-section of a solid plate is analyzed for alternative #2 where the hydraulic cylinder and the lift rod are aligned.

![Image](Figure A.28: Upper Arm cross-section by tube)

Axial Load

\[ N_{UA} = F_2 \cdot \cos(\alpha) + \frac{F_3}{2} \cdot \cos(\varphi_3) \]

Moment

\[ M_{z,UA} = F_2 \cdot x_{cp} \cdot \sin(\beta) \cdot (L_6 - L_{10}) \]
\[ M_{y,UA} = \frac{F_3}{2} \cdot \sin(\varphi_3) \cdot (L_7 - L_{10}) - F_2 \cdot \sin(\alpha) \cdot (L_6 - L_{10}) \]

Area Total

\[ A_{tot,PL} = a_{1,UA,PL} \cdot b_{1,UA,PL} \]

Point A

\[ \bar{z}_{A,UA,PL} = \frac{b_{1,UA,PL}}{2} \]
\[ \bar{y}_{A,UA,PL} = \frac{a_{1,UA,PL}}{2} \]

Moment of Inertia

\[ I_{y,UA,PL} = \frac{a_{1,UA,PL} \cdot b_{1,UA,PL}^3}{12} \]
\[ I_{z,UA,PL} = \frac{b_{1,UA,PL} \cdot a_{1,UA,PL}^3}{12} \]

Stress at A

\[ \sigma_{xx,UA,PL} = \frac{N_{UA}}{A_{tot,PL}} + \frac{M_{y,UA} \cdot \bar{z}_{A,UA,PL}}{I_{y,UA,PL}} - \frac{M_{z,UA} \cdot \bar{y}_{A,UA,PL}}{I_{z,UA,PL}} \]
Cross-Section I

The cross-section of an I-beam is analyzed for alternative #2 where the hydraulic cylinder and the lift rod are aligned.

Axial Load
\[ N_{UA} = F_2 \cdot \cos(\alpha) + \frac{F_3}{2} \cdot \cos(\varphi_3) \]

Moment
\[ M_{2,UA} = F_{2,xyz} \cdot \sin(\beta) \cdot (I_6 - I_{10}) \]
\[ M_{y,UA} = \frac{F_3}{2} \cdot \sin(\varphi_3) \cdot (I_7 - I_{10}) - F_2 \cdot \sin(\alpha) \cdot (I_6 - I_{10}) \]

Area Total
\[ A_1, UA, I = A_3, UA, I = a_1, UA, I \cdot b_1, UA, I \]
\[ A_2, UA, I = a_2, UA, I \cdot b_2, UA, I \]
\[ A_{total}, UA, I = A_1, UA, I \cdot 2 + A_2, UA, I \]

Point A
\[ z_A, UA, I = \frac{-b_2, UA, I - a_1, UA, I}{2} \]
\[ y_A, UA, I = \frac{-b_1, UA, I}{2} \]

Centroid Distance
\[ e_1, UA, I = \frac{b_2, UA, I + a_1, UA, I}{2} \]

Moment of Inertia
\[ I'_{x, UA, I} = \left( \frac{b_1, UA, I \cdot a_3^2, UA, I}{12} + e_1^2, UA, I \cdot A_1, UA, I \right) \cdot 2 + \frac{a_2, UA, I \cdot b_3^2, UA, I}{12} \]
\[ I_{z, UA, I} = \frac{a_1, UA, I \cdot b_3^2, UA, I}{12} + \frac{b_2, UA, I \cdot a_3^2, UA, I}{12} \]

Tension
\[ \sigma_{xx, UA, I} = \frac{N_{UA}}{A_{total}, UA, I} + \frac{M_{y, UA} \cdot z_A, UA, I}{I'_{x, UA, I}} - \frac{M_{z, UA} \cdot y_A, UA, I}{I_{z, UA, I}} \]

Shear at A
\[ \tau_{xy, UA, I} = 0 \]

Principal Stresses
\[ \sigma_{1, UA, I} = \frac{\sigma_{xx, UA, I}}{2} + \sqrt{\left( \frac{\sigma_{xx, UA, I}}{2} \right)^2 + \tau_{xy, UA, I}^2} \]
\[ \sigma_{2, UA, I} = \frac{\sigma_{xx, UA, I}}{2} - \sqrt{\left( \frac{\sigma_{xx, UA, I}}{2} \right)^2 + \tau_{xy, UA, I}^2} \]

Equivalent Stress
\[ \sigma_{eff, UA, I} = \sigma_{1, UA, I} - \sigma_{2, UA, I} \]

Figure A.29: A rough approx of cross-section by tube
Cross-Section T

The cross-section of a T-beam is analyzed for alternative #2 where the hydraulic cylinder and the lift rod are aligned.

Axial Load

\[ N_{UA} = F_2 \cdot \cos(\alpha) + \frac{F_3}{2} \cdot \cos(\phi_3) \]

Moment

\[ M_{z,UA} = F_2 \cdot x_{yz} \cdot \sin(\beta) \cdot (L_6 - L_{10}) \]

\[ M_{y,UA} = \frac{F_3}{2} \cdot \sin(\phi_3) \cdot (L_7 - L_{10}) - F_2 \cdot \sin(\alpha) \cdot (L_6 - L_{10}) \]

Area Total

\[ A_{i,UA,T} = a_{i,UA,T} \cdot b_{i,UA,T} \text{ for } i = 1..3 \]

Centroid Distance

\[ e_1, T = \frac{a_{1,UA,T}}{2} \]

\[ e_2, T = a_{1,UA,T} + \frac{b_{2,UA,T}}{2} \]

\[ e_3, T = a_{1,UA,T} + b_{2,UA,T} + \frac{a_{3,UA,T}}{2} \]

\[ e_z, T = \frac{\sum_{i=1}^{3} e_i, H \cdot A_i, UA, T}{A_{tot,UA,T}} \]

Point A

\[ z_{A,UA,T} = -e_2, T \]

\[ y_{A,UA,T} = -\frac{b_{1,UA,T}}{2} \]

Moment of Inertia

\[ I_{y,UA,T} = \left( \frac{b_{1,UA,T} \cdot a_{1,UA,T}^3}{12} + (e_2, T - e_1, T)^2 \cdot A_{1,UA,T} \right) \]

\[ + \left( \frac{a_{2,UA,T} \cdot b_{2,UA,T}^3}{12} + (e_2, T - e_2, T)^2 \cdot A_{2,UA,T} \right) \]

\[ + \left( \frac{b_{3,UA,T} \cdot a_{3,UA,T}^3}{12} + (e_2, T - e_3, T)^2 \cdot A_{3,UA,T} \right) \]

\[ I_{z,UA,T} = \frac{a_{1,UA,T} \cdot b_{1,UA,T}^3}{12} + \frac{b_{2,UA,T} \cdot a_{2,UA,T}^3}{12} \]

\[ + \frac{a_{3,UA,T} \cdot b_{3,UA,T}^3}{12} \]

Stress at A

\[ \sigma_{xx,UA,T} = \frac{N_{UA}}{A_{tot,UA,T}} + \frac{M_{z,UA} \cdot z_{A,UA,T}}{I_{y,UA,T}} + \frac{M_{y,UA} \cdot y_{A,UA,T}}{I_{z,UA,T}} \]
Cross-Section I2

The existing structure is analyzed for alternative #1 where there is a torque \((M_{x,UA})\) action on the structure due to not aligning the Lift Rod with the hydraulic cylinder.

Axial Load (x-axis)

\[
N_{UA} = F_2 \cdot \cos(\alpha) + \frac{F_3}{2} \cdot \cos(\varphi_3)
\]

Moment

\[
M_{x,UA} = F_2 \cdot \sin(\alpha) \cdot L_6, \\
M_{y,UA} = -F_2 \cdot \cos(\alpha) \cdot L_6
\]

Area Total Point A

\[
A_{tot,12} = a_1, UA, \cdot b_{1,1}, UA, \cdot I^3 + a_2, UA, \cdot b_{2,1}, UA, \cdot I^2
\]

\[
Z_{A,12} = \frac{b_{1,1}, UA, I_2}{2}
\]

\[
y_{A,12,12} = \frac{b_{2,1}, UA, I_2}{2} + \frac{3}{2} \cdot a_1, UA, I_2
\]

Moment of Inertia

\[
I_{y,12} = \frac{a_{1,1,12} \cdot b_{3,1,12}^3}{12} + \frac{b_{2,1,12} \cdot a_{2,1,12}^3 \cdot UA, I_2}{12} \cdot I_2
\]

\[
I_{z,12} = \frac{a_{2,1,12} \cdot b_{3,1,12}^3}{12}
\]

\[
+ \left( \frac{a_{2,1,12} \cdot b_{3,1,12}^3}{12} + \left( \frac{b_{2,1,12} \cdot a_{2,1,12}^3 \cdot UA, I_2}{2} \right)^2 \cdot a_{2,1,12} \cdot b_{2,1,12} \right)^2
\]

\[
+ \left( \frac{b_{2,1,12} \cdot a_{2,1,12}^3 \cdot UA, I_2}{2} \right) \cdot a_{2,1,12} \cdot b_{2,1,12}
\]

Tension Stress at A

\[
\sigma_{xx,UA,12} = \frac{N_{UA}}{A_{tot,12}} + \frac{M_{y,UA} \cdot z_{A,12}}{I_{y,12}} - \frac{M_{z,UA} \cdot y_{A,12}}{I_{z,12}}
\]

Shear at A

\[
K_{v1,UA,12} = K_{v2,UA,12} = K_{v3,UA,12} = 3 \cdot a_{1,1,12} \cdot b_{1,1,12}
\]

\[
K_{w1,UA,12} = K_{w2,UA,12} = 0.3 \cdot a_{2,1,12} \cdot b_{2,1,12}
\]

\[
W_{v1,UA,12} = 0.3 \cdot a_{2,1,12} \cdot b_{1,1,12} \cdot a_{1,1,12}
\]

\[
\tau_{xy,UA,12} = \sqrt{K_{v1,UA,12} + K_{v2,UA,12} + K_{v3,UA,12} + K_{v4,UA,12} + K_{v5,UA,12}}
\]

Principal Stresses

\[
\sigma_1,UA,12 = \frac{\sigma_{xx,UA,12}}{2} + \sqrt{\left( \frac{\sigma_{xx,UA,12}}{2} \right)^2 + \tau_{xy,UA,12}^2}
\]

\[
\sigma_2,UA,12 = \frac{\sigma_{xx,UA,12}}{2} - \sqrt{\left( \frac{\sigma_{xx,UA,12}}{2} \right)^2 + \tau_{xy,UA,12}^2}
\]

Equivalent Stress

\[
\sigma_{eff,UA,12} = \sqrt{\sigma_1,UA,12^2 - \sigma_2,UA,12^2}
\]
A.3.7 Upper Arm Tube

The tube connecting the two arms is analyzed for maximum stress scenario. Both circular tube and rectangular tube is analyzed for implementation. The tube is considered fixed at the right side arm when the full load is applied strictly on the left side arm.

**Alternative #1** with the equivalent torques acting on the arm according to figure A.33.

**Figure A.32**: Tube linking arms

**Figure A.33**: Resulting forces and moment

**Figure A.34**: Circular cross-section

**Figure A.35**: Rectangular cross-section

Resulting Forces

\[
R_{z, tu} = R_{ZR} - \frac{F_3}{2} \sin(\varphi_3)
\]

\[
R_{y, tu} = R_{XR} - \frac{F_3}{2} \cos(\varphi_3)
\]

\[
M_{x, tu} = -R_{ZR} \cdot l_{10} - \frac{F_3}{2} \sin(\varphi_3) \cdot (L_7 - L_{10})
\]

Acting Moments

\[
M_{y, tu} = -R_{z, tu} \cdot L_9
\]

\[
M_{z, tu} = -R_{y, tu} \cdot L_9
\]
Circular Cross-Section

Moment of Inertia
\[ I_y^c = \frac{\pi \cdot \left(d_c^4 - \left(d_c - 2 \cdot t_c\right)^4\right)}{64} \]
\[ I_z^c = \frac{\pi \cdot \left(d_c^4 - \left(d_c - 2 \cdot t_c\right)^4\right)}{64} \]

Area
\[ A_c = \frac{d_c^2 \cdot \pi}{4} \]

Section Modulus in Torsion
\[ W_{v,c} = \frac{\pi \cdot d_c^2 \cdot t_c}{2} \]

Point A
\[ z_{A,c} = \frac{d_c}{2 \sqrt{2}} \]
\[ y_{A,c} = \frac{d_c}{2 \sqrt{2}} \]
\[ S_{A,A,c} = \left(\frac{d_c^2 \cdot \pi}{4} - \frac{(d_c - t_c)^2 \cdot \pi}{4}\right) \cdot \frac{1}{4} \cdot \left(\frac{2 + \sqrt{2}}{4}\right) \cdot d_c \] (very rough approx)
\[ \tau_{xy,A,c} = \frac{M_{x,y}}{W_{v,c}} + \frac{R_{y,mb} \cdot S_{A,A,c}}{I_z \cdot t_c} + \frac{R_{z,mb} \cdot S_{A,A,c}}{I_x \cdot t_c} \]
\[ \sigma_{xx,A,c} = \frac{M_{x,y} \cdot z_{A,c}}{I_y^c} - \frac{M_{x,y} \cdot y_{A,c}}{I_x^c} \]
\[ \sigma_{1,A,c} = \frac{\sigma_{xx,A,c}}{2} + \sqrt{\left(\frac{\sigma_{xx,A,c}}{2}\right)^2 + \tau_{xy,A,c}^2} \]
\[ \sigma_{2,A,c} = \frac{\sigma_{xx,A,c}}{2} - \sqrt{\left(\frac{\sigma_{xx,A,c}}{2}\right)^2 + \tau_{xy,A,c}^2} \]
\[ \sigma_{eff,A,c} = \sigma_{1,A,c} - \sigma_{2,A,c} \]

Point B
\[ z_{B,c} = \frac{d_c}{2} \]
\[ S_{A,B,c} = \left(\frac{d_c^2 \cdot \pi}{4} - \frac{(d_c - t_c)^2 \cdot \pi}{4}\right) \cdot \frac{1}{2} \cdot \frac{d_c}{\sqrt{2}} \] (rough approx)
\[ \tau_{xy,B,c} = \frac{M_{x,y}}{W_{v,c}} + \frac{R_{y,mb} \cdot S_{A,B,c}}{I_z \cdot t_c} \]
\[ \sigma_{xx,B,c} = \frac{M_{x,y} \cdot z_{B,c}}{I_y^c} \]
\[ \sigma_{1,B,r} = \frac{\sigma_{xx,B,r}}{2} + \sqrt{\left(\frac{\sigma_{xx,B,r}}{2}\right)^2 + \tau_{xy,B,r}^2} \]
\[ \sigma_{2,B,r} = \frac{\sigma_{xx,B,r}}{2} - \sqrt{\left(\frac{\sigma_{xx,B,r}}{2}\right)^2 + \tau_{xy,B,r}^2} \]
\[ \sigma_{eff,B,c} = \sigma_{1,B,c} - \sigma_{2,B,c} \]
Rectangular Cross-Section

Moment of Inertia

\[ I_y = \frac{(b_r \cdot a_r - (b_r - 2 \cdot t_r) \cdot (a_r - 2 \cdot t_r))^3}{12} \]
\[ I_z = \frac{(a_r \cdot b_r - (a_r - 2 \cdot t_r) \cdot (b_r - 2 \cdot t_r))^3}{12} \]

Area

\[ A = a_r \cdot b_r - (a_r - 2 \cdot t_r) \cdot (b_r - 2 \cdot t_r) \]

Section Modulus in Torsion

\[ W_{v,r} = 2 \cdot A \cdot t_r \]

Point A

\[ z_{A,r} = \frac{a_r}{2} \]
\[ y_{A,r} = \frac{b_r}{2} \]
\[ \tau_{xy,A,r} = \frac{M_{x,ru}}{W_{v,r}} \]
\[ \sigma_{xx,A,r} = \frac{M_{y,ru} \cdot z_{A,r}}{I_y} \]
\[ \sigma_{1, A,r} = \frac{\sigma_{xx,A,r}}{2} + \sqrt{\left( \frac{\sigma_{xx,A,r}}{2} \right)^2 + \tau_{xy,A,r}^2} \]
\[ \sigma_{2, A,r} = \frac{\sigma_{xx,A,r}}{2} - \sqrt{\left( \frac{\sigma_{xx,A,r}}{2} \right)^2 + \tau_{xy,A,r}^2} \]
\[ \sigma_{eff,A,r} = \sigma_{1, A,r} - \sigma_{2, A,r} \]

Point B

\[ z_{B,r} = \frac{a_r}{2} \]
\[ S_{A,B,r} = \left( \frac{b_r}{2} \cdot t_r + \frac{b_r}{4} \right) \cdot 2 + a_r \cdot t_r \]
\[ \tau_{xy,B,r} = \frac{M_{x,ru}}{W_{v,r}} + \frac{R_{y,ru} \cdot S_{A,B,r}}{I_z \cdot t_r} \]
\[ \sigma_{xx,B,r} = \frac{M_{y,ru} \cdot z_{B,r}}{I_y} \]
\[ \sigma_{1, B,r} = \frac{\sigma_{xx,B,r}}{2} + \sqrt{\left( \frac{\sigma_{xx,B,r}}{2} \right)^2 + \tau_{xy,B,r}^2} \]
\[ \sigma_{2, B,r} = \frac{\sigma_{xx,B,r}}{2} - \sqrt{\left( \frac{\sigma_{xx,B,r}}{2} \right)^2 + \tau_{xy,B,r}^2} \]
\[ \sigma_{eff,B,r} = \sigma_{1, B,r} - \sigma_{2, B,r} \]
Alternative #2 with the hydraulic cylinder and lift rod aligned on the x-axis.

Figure A.36: Tube linking arms

Figure A.37: Resulting forces and moment

Figure A.38: Circular cross-section

Figure A.39: Rectangular cross-section

Resulting Forces

\[
R_{x, tu} = \frac{R_y}{2}
\]

\[
R_{y, tu} = R_{XR} - \frac{F_3}{2} \cos(\varphi_3)
\]

\[
R_{z, tu} = R_{ZR} - \frac{F_3}{2} \sin(\varphi_3)
\]

\[
M_{x, tu} = -R_{ZR} \cdot L_{10} - \frac{F_3}{2} \sin(\varphi_3) \cdot (L_7 - L_{10})
\]

Acting

\[
M_{y, tu} = -R_{z, tu} \cdot L_9
\]

Moments

\[
M_{z, tu} = -R_{y, tu} \cdot L_9
\]
Circular Cross-Section

Moment of Inertia
\[ I_y = \frac{\pi \cdot (d_c^4 - (d_c - 2 \cdot t_c)^4)}{64} \]
\[ I_z = \frac{\pi \cdot (d_c^4 - (d_c - 2 \cdot t_c)^4)}{64} \]

Area
\[ A_c = \frac{d_c^2 \cdot \pi}{4} \]

Section Modulus in Torsion
\[ W_{v,c} = \frac{\pi \cdot d_c^2 \cdot t_c}{2} \]

Point A
\[ z_{A,c} = -\frac{d_c}{2 \cdot \sqrt{2}} \]
\[ y_{A,c} = \frac{d_c}{2 \cdot \sqrt{2}} \]

\[ S_{A,A,c} = \frac{d_c^2 \cdot \pi}{4} - \frac{(d_c - t_c)^2 \cdot \pi}{4} \cdot \frac{1}{4} \left( \frac{2 + \sqrt{2} \cdot d_c}{4} \right) \]
\[ \tau_{xy, A,c} = \frac{R_{x, tu} \cdot S_{A,A,c}}{W_{v,c}} + \frac{R_{z, tu} \cdot S_{A,A,c}}{I_z \cdot t_c} \]
\[ \sigma_{xx, A,c} = \frac{R_{x, tu} \cdot z_{A,c}}{A_c} + \frac{M_{x, tu} \cdot z_{A,c}}{I_y} - \frac{M_{z, tu} \cdot y_{A,c}}{I_z} \]
\[ \sigma_{1, A,c} = \frac{\sigma_{xx, A,c}}{2} + \sqrt{\left( \frac{\sigma_{xx, A,c}}{2} \right)^2 + \tau_{xy, A,c}^2} \]
\[ \sigma_{2, A,c} = \frac{\sigma_{xx, A,c}}{2} - \sqrt{\left( \frac{\sigma_{xx, A,c}}{2} \right)^2 + \tau_{xy, A,c}^2} \]
\[ \sigma_{eff, A,c} = \sigma_{1, A,c} - \sigma_{2, A,c} \]

Point B
\[ z_{B,c} = -\frac{d_c}{2} \]

\[ S_{A,B,c} = \frac{d_c^2 \cdot \pi}{4} - \frac{(d_c - t_c)^2 \cdot \pi}{4} \cdot \frac{1}{2 \cdot \sqrt{2}} \]
\[ \tau_{xy, B,c} = \frac{M_{x, tu} \cdot S_{A,B,c}}{W_{v,c}} + \frac{R_{z, tu} \cdot S_{A,B,c}}{I_z \cdot t_c} \]
\[ \sigma_{xx, B,c} = \frac{R_{x, tu} \cdot z_{B,c}}{A_c} + \frac{M_{x, tu} \cdot z_{B,c}}{I_y} \]
\[ \sigma_{1, B,c} = \frac{\sigma_{xx, B,c}}{2} + \sqrt{\left( \frac{\sigma_{xx, B,c}}{2} \right)^2 + \tau_{xy, B,c}^2} \]
\[ \sigma_{2, B,c} = \frac{\sigma_{xx, B,c}}{2} - \sqrt{\left( \frac{\sigma_{xx, B,c}}{2} \right)^2 + \tau_{xy, B,c}^2} \]
\[ \sigma_{eff, B,c} = \sigma_{1, B,c} - \sigma_{2, B,c} \]
Rectangular Cross-Section

Moment of Inertia
\[ I_y = \frac{b_r \cdot a_r^3 - (b_r - 2 \cdot t_r) \cdot (a_r - 2 \cdot t_r)^3}{12} \]
\[ I_z = \frac{(a_r \cdot b_r^3 - (a_r - 2 \cdot t_r) \cdot (b_r - 2 \cdot t_r)^3}{12} \]

Area
\[ A_r = a_r \cdot b_r - (a_r - 2 \cdot t_r) \cdot (b_r - 2 \cdot t_r) \]

Section Modulus in Torsion
\[ W_{v,r} = 2 \cdot A_r \cdot t_r \]

Point A
\[ z_A \cdot r = \frac{a_r}{2} \]
\[ y_A \cdot r = \frac{b_r}{2} \]
\[ \tau_{xy, A} = \frac{M_{x, A}}{W_{v, r}} \]
\[ \sigma_{xx, A} = \frac{R_{x, A} + M_{y, A} \cdot z_A \cdot r}{A_r} - \frac{M_{z, A} \cdot y_A \cdot r}{I_z} \]
\[ \sigma_{1, A} = \frac{\sigma_{xx, A} + \sqrt{\left( \frac{\sigma_{xx, A}}{2} \right)^2 + \tau_{xy, A}^2}}{2} \]
\[ \sigma_{2, A} = \frac{\sigma_{xx, A} - \sqrt{\left( \frac{\sigma_{xx, A}}{2} \right)^2 + \tau_{xy, A}^2}}{2} \]
\[ \sigma_{eff, A} = \sigma_{1, A} - \sigma_{2, A} \]

Point B
\[ z_B \cdot r = \frac{a_r}{2} \]
\[ S_{A, B} = \left( \frac{b_r \cdot t_r \cdot b_r \cdot t_r}{4} \right)^2 + a_r \cdot t_r \cdot \frac{b_r}{2} \]
\[ \tau_{xy, B} = \frac{M_{x, B} + R_{z, B} \cdot S_{A, B}}{W_{v, r}} \]
\[ \sigma_{xx, B} = \frac{R_{z, B}}{A_r} + \frac{M_{y, B} \cdot z_B \cdot r}{I_y} \]
\[ \sigma_{1, B} = \frac{\sigma_{xx, B} + \sqrt{\left( \frac{\sigma_{xx, B}}{2} \right)^2 + \tau_{xy, B}^2}}{2} \]
\[ \sigma_{2, B} = \frac{\sigma_{xx, B} - \sqrt{\left( \frac{\sigma_{xx, B}}{2} \right)^2 + \tau_{xy, B}^2}}{2} \]
\[ \sigma_{eff, B} = \sigma_{1, B} - \sigma_{2, B} \]
Appendix B
B.1  Positioning of Pivoting Points

B.1.1  Lower Link

PTO Distance
The distance from the PTO shaft to the Lower Hitch Points when the lower links are in a horizontal plane level to the ground is specified by ASAE. The range is set to 710-775mm for category 4H with a PTO shaft diameter of 45mm [4]. This range was recently converted to match ISO standard where the old ASAE Standard was shorter. In order to be compatible with both new implements as well as older implements for the North American market, ASAE recommends designing for the minimum value in the specified range.

The location of the PTO shaft is preset and cannot be altered. Therefore, adjustment can only be made by moving the attachment of the Lower Link, LLP, which is constrained by the clearance to the Hydraulic cylinder. This results in that the minimum PTO distance must be compromised to keep the length of the Lower Link within reach of the constraint for transport height.

The current version has a PTO distance of 717 which is considered sufficient for the current market.

It is concluded stretch the PTO Distance slightly but still to be designed within the shorter region of the standardized range. It is set to 730mm.

Vertical Gain
The raise of the Lower Link Hitch Points in a vertical plane perpendicular to the ground is illustrated by the curve for sin(x). Figure B.1 illustrates that the vertical gain is relatively linear up to 45° where the curve levels out. The curve for horizontal gain is inverted were the horizontal gain starts of small and increases more rapidly after 30°. It levels out to linear at a maximum gain after 45°. It is desired to have optimal vertical gain and minimum horizontal movement. The horizontal gain must however be slightly compromised due to the required angle for top positioning.

In conclusion the lower link at the top positioning is set not exceed 40° relative to the ground. This to meet with the required transport height with a maximized vertical gain throughout the raise. The minimization of the horizontal movement is slightly compromised to meet with this constraint.
Positioning of the attachment to the rear frame

The main factors to consider for the positioning of the lower link are the minimum transport height 1200mm, the max lower positioning 230mm and the movement range min 900mm. All these measurements are increased by moving the attachment point, LLP, further away from the hitch points (the hitch point location is preset from the PTO Distance). By so, the Lower Link gets longer and will have a greater vertical gain per angle pivoted. Also this will minimize the movement horizontal to the ground for a more consistent mast pitch throughout the raise. The length of the lower link is however constrained by the interference with the Hydraulic Cylinder.

The distance for the Hydraulic Cylinder positioning, HCP, in a horizontal plane to the ground relative to the PTO is preset to 215mm (UA, is preset in accordance with section 7.3.2 Hydraulic Cylinder).

This results in a distance of 945mm between the Hydraulic Cylinder attachment, HCP, and the Lower Link Hitch Point, LLHP.

Constraints for the Transport Height are set with the following values:

- The variable $x_1$ is set as the distance of LLP away from the Hydraulic Cylinder, HCP, in a horizontal plane to the ground.
- Angle of the lower link in the fully raised position is set to 40°.
- The distance from LLHP to HCP is set to 945mm (215+730).
- The distance of 190mm between LLP and HCP is set to give sufficient clearance when $x_1=0$.
- A distance of 1225mm is estimated to be a sufficient target for the minimum Transport Height.

This results in the following equations:

\[
\begin{align*}
\text{The distance } y_1 \text{ of LLP above the ground:} & \quad y_1 = 1225 - (x_1 + 945)\sin(40) \\
\text{The distance } y_2 \text{ of HCP above the ground:} & \quad y_2 = y_1 + 190 + x_2 \cdot \tan(40) \quad \Rightarrow \quad y_2 = 0.2 x_1 + 805
\end{align*}
\]

Note that this is approximated in 2D and is only considered as a reference for initial modeling.

Equation for $y_2$ shows that minimizing $x_1$, minimizes the distance $y_2$ (for positive values of $x_1$). By so, the gain of stroke for the Hydraulic Cylinder is optimized. However, the optimization of the Hydraulic Cylinder stroke comes with the cost of compromising the Lower Link length resulting in greater unwanted horizontal gain.

The distance of LLP above the ground is preferred to be around half the Transport Height. Analyzing the constraints, a positioning where $x_1$ equals 30mm is considered to be a good compromise for the location. This results in a positioning of LLP 600mm above the ground, approximately midway to the required transport height. For this, the length of the Lower Link results in 975mm.

For the convergence, the design is to be optimized for category 4, but remain interchangeable with
category 4N. It is assumed that a convergence in the mid range the specified standard is optimal. Therefore, with Lower Link length set to 975mm, a convergence of 2400mm results in a Lower Link Point span of 830mm. For category 4N the convergence results in 8900mm, outside of the range but considered acceptable.

In conclusion LLP is constrained by the standardized measurements and the positioning relative the hydraulic cylinder. It is set to be located 30 mm away from the hydraulic cylinder resulting in the distance $y_1=600$mm above the ground. The Lower Link length is set to the resulting distance 975mm. The convergence is set 2400mm resulting in a Lower Link Hitch Point span of 830mm. Modeling gives the tire clearance at a fully raised position to a sufficient distance of 300mm. The coordinates of LLP is set as the origin of the linkage system:

$$LL_p = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
B.1.2 Hydraulic Cylinder

Positioning
The positioning of the bottom attachment away from the rear frame is set by the clearance at the scenario of the Hydraulic Cylinder is fully extended. At this point, the angle of the cylinder is set to be parallel with the rear frame. For the clearance, the outer cylinder is estimated not to have a greater outer diameter then 130mm. This in conjunction with an estimated sufficient clearance of 10mm to the attachment plate of 12.7mm gives a positioning in a horizontal plane of 90mm away from the rear frame.

For the vertical positioning, placing the Hydraulic Cylinder Point, HC_P, lower increases the possible stroke of the Hydraulic Cylinder. The minimum clearance to the lower link is set to 190mm for a 19.1mm thick plate dividing the two components. The distance in a vertical plane for sufficient clearance is set to 190+x_1sin(40) resulting in a distance of 209mm.

Stroke
The stroke of the hydraulic cylinder is constrained by its fixed length. From there, by extending the retracted length; the stroke can be increased on a ratio of 1/1. The retracted length is constrained by the clearance in between the Lower Link and the Upper Arm.

The stroke is illustrated by distance h_5 in figure B.3. All other measurements are fixed and initially estimated as follows:

<table>
<thead>
<tr>
<th>Fixed Distances:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>h_1=41.3mm</td>
<td>bottom attachment</td>
</tr>
<tr>
<td>h_2=12.7mm</td>
<td>bottom plate</td>
</tr>
<tr>
<td>h_3=10mm</td>
<td>piston nut</td>
</tr>
<tr>
<td>h_4=50mm</td>
<td>piston</td>
</tr>
<tr>
<td>h_5=55mm</td>
<td>cylinder head</td>
</tr>
<tr>
<td>h_6=116mm</td>
<td>top attachment</td>
</tr>
</tbody>
</table>

= 285

In section B.1.3, h_5 is set to 335mm resulting in:

| HC_0=285+h_5        | 0% extended |
| HC_100=285+2h_5     | 100% extended |
| HC_0=620mm          | 0% extended |
| HC_100=955mm        | 100% extended |

In conclusion the Hydraulic Cylinder is positioned as close to the rear frame possible and as low as possible. This optimizes the stroke for Movement Range and the Upper Arm angle when fully lowered for the Lower Hitch Point Height. The positioning in a horizontal plane to the ground is set to 90mm away from the rear frame. The vertical distance for clearance is set for 209mm. This gives the following coordinates relative to LLP:

\[
HC_P = \begin{bmatrix} 30 \\ 209 \end{bmatrix}
\]
B.1.3 Upper Arm

Positioning of Upper Arm Attachment

The moving pattern of the Upper Arm increases by locating the pivot point of the attachment closer to the hydraulic cylinders in a horizontal plane and approximately at the midway cylinder extension in a vertical plane. For the angle of the Upper Arm fully raised, even if the vertical gain decreases after 45° of raise, the Lower Hitch Points still gains vertical raise since the horizontal movement of the upper arm also lifts the Lower Links. The fully raised positioning is however constrained by the clearance to the lift rod clevis attachment. This constraint limits the fully raised angle of the Upper Arm around 45°. To satisfy the constraint of the Movement Range, the attachment must be located high enough to give a sufficient down angle at a fully lowered positioning. The positioning of Upper Arm is also constrained by the clearance of the Upper Arm tube to a coupler on top of the PTO. A minimum distance of 325mm to the Upper Arm attachment, UAP, is considered sufficient.

Parameters for the Upper Arm positioning, UAP, is set as follows. The attachment is to be located to give the Upper Arm an angle of 45° degrees at a fully raised position and to have sufficient down angle at a fully lowered position. Iterative testing in conjunction with length data from section 7.3.8 (Lift Rod) gives an angle of 20° to be sufficient. The hydraulic cylinder is set to have a fully vertical position when fully extended. The distance from the Upper Arm attachment to the Hydraulic Cylinder, UAHC, is set to 325mm.

Data:

\[ HC_x = 30 \]
\[ HC_y = 209 \]
\[ UAHC_L = 325 \]
\[ w_1 = 45^\circ \]
\[ w_2 = 20^\circ \]

Hydraulic Cylinder length is given as a function of stroke, variable \( h_5 \):

\[ HC_0 = 285 + h_5 \]
\[ HC_{100} = 285 + 2 \cdot h_5 \]

Equations:

\[ UA_x = HC_x - UAHC_L \cdot \cos(w_1) \]
\[ UA_y = HC_y + HC_{100} - UAHC_L \cdot \sin(w_1) \]
\[ UAHC_{0,x} = UA_x + UAHC_L \cdot \cos(w_2) \]
\[ UAHC_{0,y} = UA_y - UAHC_L \cdot \sin(w_2) \]

\[ (UAHC_{0,x} - HC_x)^2 + (UAHC_{0,y} - HC_y)^2 = HC_0^2 \]

Equations solved for positive stroke gives:

\[ UA_x = -200 \] Coordinates relative LLP
\[ UA_y = 935 \] Cylinder Stroke
\[ h_5 = 335 \]

In conclusion the upper arm attachment coordinates are set to, \( \begin{bmatrix} UA_x \\ UA_y \end{bmatrix} \). The Hydraulic Cylinder Stroke \( h_5 \) is set to 335mm.
B.1.4 Lift Rod

Moving Pattern
Basic testing for the Movement Range depending on the length of the Lift Rod gives the movement range to increase for greater length of the Lift Rod. **In conclusion the Movement range is to be set for constrains with the scenario of the Lift Rod fully shortened.**

Positioning for the attachment of the Lift Rod
Optimal stress distribution on the Lift Rod occurs when the Lift Rod attachment to the Lower Link, LLLR, is aligned with the Lower Link Hitch Point and the attachment to the Upper Arm, UALR, is aligned with the Hydraulic Cylinder attachment, UAHC. The optimal stress distribution is however compromised for gain of required movement range. The attaching points of the Lift Rod are therefore optimized for stress and weight reduction with the constraint of the movement pattern. For this compromise, the system is analyzed with the distances UAP to UALR ($L_6$) and LLP to LLLR ($L_2$) set as variables.

Stress optimization was conducted with varying geometry of the pivoting points of the Lift Rod. The target function was to minimize the sum of the forces acting on the key components. Note that the force summation acting on the linkage may not be optimal from a stress point of view for the lower link and the upper arm. These two components both have critical points due to resulting bending stress.
B.2 Force and Stress Analysis

Forces

Force $F_1$ acting on the Upper Link

$$F_1 = \frac{P L_4 \cos(\theta)}{L_3 \left( \cos(\theta) \cos(\phi_2) + \sin(\phi_2) \sin(\theta) \right)}$$

Force $F_2$ acting on the Lift Rod

$$F_2 = \left( P L_1 \left( \cos(\phi_1) L_4 \cos(\theta) \sin(\phi_2) + \cos(\phi_1) \sin(\phi_2) L_4 \cos(\theta) \right) + \cos(\phi_1) \sin(\phi_2) L_4 \sin(\theta) \cos(\phi_2) + \sin(\phi_1) \cos(\phi_2) \sin(\theta) \right) /$$

$$\left( L_2 L_5 \left( \sin(\alpha) \cos(\phi_1) \cos(\phi_2) \cos(\theta) + \sin(\alpha) \cos(\phi_2) \sin(\theta) \right) + \cos(\alpha) \sin(\phi_1) \cos(\phi_2) \cos(\theta) + \cos(\alpha) \sin(\phi_2) \sin(\theta) \right)$$

Force $F_3$ acting on the Hydraulic Cylinders

$$F_3 = \left( P L_1 L_6 \left( \sin(\alpha) \cos(\phi_4) \cos(\phi_2) L_4 \cos(\theta) \sin(\phi_2) \right. \right.$$

$$\left. + \sin(\alpha) \cos(\phi_4) \cos(\phi_2) \sin(\phi_2) L_4 \cos(\theta) \right)$$

$$\left. + \sin(\alpha) \cos(\phi_4) \cos(\phi_2) \sin(\phi_2) L_4 \sin(\theta) \right)$$

$$\left. - \sin(\alpha) \cos(\phi_4) L_4 \cos(\theta) \cos(\phi_2) \sin(\theta) \right) /$$

$$\left( L_7 L_5 \left( \sin(\phi_4) \cos(\phi_2) \sin(\alpha) \cos(\phi_2) \cos(\theta) \right. \right.$$

$$\left. + \sin(\phi_4) \cos(\phi_2) \sin(\alpha) \cos(\phi_2) \sin(\theta) \right)$$

$$\left. + \sin(\phi_4) \cos(\phi_2) \cos(\alpha) \sin(\phi_2) \cos(\theta) \right)$$

$$\left. + \cos(\alpha) \sin(\phi_2) \sin(\phi_2) \cos(\theta) \right)$$

$$\left. - \cos(\phi_4) \sin(\phi_2) \sin(\alpha) \cos(\phi_2) \cos(\theta) \right)$$

$$\left. - \cos(\phi_4) \sin(\phi_2) \sin(\alpha) \cos(\phi_2) \sin(\theta) \right)$$

$$\left. - \cos(\phi_4) \sin(\phi_2) \cos(\alpha) \sin(\phi_2) \cos(\theta) \right)$$

$$\left. - \cos(\phi_4) \sin(\phi_2) \cos(\alpha) \sin(\phi_2) \sin(\theta) \right) \right)$$

Force $R$ that acts at Upper Arm Attachment is described in the following simplified scenario.
Simplified Scenario

The scenario is simplified to a single loading scenario with the Lower Links and the upper arm level to the ground. The variables are set as follows:

\[ \begin{align*}
\varphi_1 &= 0 \\
\varphi_2 &= 10 \\
\varphi_3 &= 0 \\
\varphi_4 &= 0 \\
\theta &= 0 \\
\beta &= 0 \\
\end{align*} \]

Length [mm]
\[ \begin{align*}
L_1 &= 975 \\
L_2 &= 685 \\
L_3 &= 610 \\
L_4 &= 340 \\
L_5 &= 740 \\
\end{align*} \]

Force [N]
\[ P = 88960 \quad (20,000 \text{lb}) \]

Resulting Forces [N]:
\[ \begin{align*}
F_1 &= 1.01438788010^5 \quad \text{Not affected by } L_2 \text{ or } L_6 \Rightarrow \text{neglected in target function} \\
F_2 &= \frac{1.03910294210^8}{L_2 \sin(\alpha)} \\
F_3 &= \frac{3.05618512210^5 L_6}{L_2} \\
R_L &= \frac{1}{2} \sqrt{\frac{4.312935692 \times 10^5 \cos(\alpha)^2}{\sin(\alpha)^2 L_2^2} + \frac{0.3293175823 \left(3.621458471 \times 10^5 \sin(\alpha) - 5.325644010 \times 10^5 \sin(\alpha) L_6 \right)^2}{L_2^2 \sin(\alpha)^2}} \\
RR &= \frac{1.52809256210^5 \cdot L_6}{L_2} \\
R &= \frac{R_L + RR}{2} \quad \text{A very rough value of an average per side} \\
\end{align*} \]

Figure B.5: Simplified loading scenario for force optimization

For lengths \( L_2 \) and angle \( \beta \) not shown in figure B.5, please refer to appendix A for diagram.
**Constraints**

The Movement Range, $MR$, is set as a constraint for the optimization. The movement range is set by the distance the lower hitch points moves vertically when the hydraulic cylinder is extended from 0% to 100%. The constraint is set to 950mm where the extra 50mm is added on for losses in the substantial approximations.

$$MR = LL_{hp}(100) - LL_{hp}(0) \geq 950$$

*For calculations, please refer to appendix A.1*

**Other Constraints:**

- The distance $L_2$, must be within the Lower Link Length, $L_1$.
  $$L_2 \leq L_1$$

- The distance $L_6$, must be greater the distance to the attachment of the Hydraulic Cylinder, $L_7$.
  $$L_6 \geq L_7$$

- The angle $\alpha$ is to be kept within the range of 0-90°
  $$L_6 \leq -UA_x + L_2$$

**Optimization**

The target function, $Obj$, is minimized by the adjustable variables $L_2$ and $L_6$.

| Target Function: $Obj = F_2 + F_3 + R$ | Adjustable Variables: $L_2$, $L_6$ | Constraints: $L_2 \leq L_1$, $L_6 \geq L_7$, $L_6 \leq -UA_x + L_2$, $MR \geq 950$ |
## Results

### Table B.2: Answer Report

<table>
<thead>
<tr>
<th>Microsoft Excel Answer Report</th>
<th>Worksheet: [2d-opt3.xls]Blad1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Report Created: 2007-12-11 09:05:44</td>
<td></td>
</tr>
</tbody>
</table>

### Target Cell (Min)

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Original Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$44 Obj=R+F2+F3</td>
<td>793 126</td>
<td>480 401</td>
</tr>
</tbody>
</table>

### Adjustable Cells

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Original Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$54 LLLR</td>
<td>325</td>
<td>975</td>
</tr>
<tr>
<td>$B$55 UALR</td>
<td>325</td>
<td>909,476166</td>
</tr>
</tbody>
</table>

### Constraints

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Cell Value</th>
<th>Formula</th>
<th>Status</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$48 L2 &lt;= L1</td>
<td>975</td>
<td>$B$48&lt;=$D$48</td>
<td>Binding</td>
<td>0</td>
</tr>
<tr>
<td>$B$49 L6 =&gt; L7</td>
<td>909,476166</td>
<td>$B$49&gt;=$D$49</td>
<td>Not Binding</td>
<td>584,47617</td>
</tr>
<tr>
<td>$B$50 L6&lt;L7+Uax</td>
<td>909,476166</td>
<td>$B$50&lt;=$D$50</td>
<td>Not Binding</td>
<td>265,52383</td>
</tr>
<tr>
<td>$B$51 Movement Range</td>
<td>950,00</td>
<td>$B$51&gt;=$D$51</td>
<td>Binding</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table B.3: Sensitivity Report

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Report Created: 2007-12-11 09:05:44</td>
<td></td>
</tr>
</tbody>
</table>

### Adjustable Cells

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Final Value</th>
<th>Reduced Gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$54 LLLR</td>
<td>975</td>
<td>0</td>
</tr>
<tr>
<td>$B$55 UALR</td>
<td>909,47617</td>
<td>0</td>
</tr>
</tbody>
</table>

### Constraints

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Final Value</th>
<th>Lagrange Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$48 L2 &lt;= L1</td>
<td>975</td>
<td>-81,7095324</td>
</tr>
<tr>
<td>$B$49 L6 =&gt; L7</td>
<td>909,47617</td>
<td>0</td>
</tr>
<tr>
<td>$B$50 L6&lt;L7+Uax</td>
<td>909,47617</td>
<td>0</td>
</tr>
<tr>
<td>$B$51 Movement Range</td>
<td>950,00</td>
<td>418,89</td>
</tr>
</tbody>
</table>
Data for optimization analysis

Resulting Data with the Movement Range, MR, kept constant at 950mm.

**Table B.4: Data for plots in figure B.6 and figure B.7**

<table>
<thead>
<tr>
<th>Loading Scenario</th>
<th>L1 [mm]</th>
<th>L2 [mm]</th>
<th>R [kN]</th>
<th>F2 [kN]</th>
<th>F3 [kN]</th>
<th>Obj [kN]</th>
<th>MR [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>975</td>
<td>969</td>
<td>94</td>
<td>110</td>
<td>274</td>
<td>480</td>
<td>950</td>
</tr>
<tr>
<td>2</td>
<td>875</td>
<td>816</td>
<td>91</td>
<td>123</td>
<td>275</td>
<td>490</td>
<td>950</td>
</tr>
<tr>
<td>3</td>
<td>775</td>
<td>723</td>
<td>89</td>
<td>138</td>
<td>275</td>
<td>504</td>
<td>950</td>
</tr>
<tr>
<td>4</td>
<td>675</td>
<td>629</td>
<td>92</td>
<td>159</td>
<td>275</td>
<td>527</td>
<td>950</td>
</tr>
<tr>
<td>5</td>
<td>575</td>
<td>536</td>
<td>101</td>
<td>186</td>
<td>276</td>
<td>564</td>
<td>950</td>
</tr>
<tr>
<td>6</td>
<td>475</td>
<td>443</td>
<td>118</td>
<td>225</td>
<td>276</td>
<td>620</td>
<td>950</td>
</tr>
<tr>
<td>7</td>
<td>375</td>
<td>349</td>
<td>146</td>
<td>285</td>
<td>276</td>
<td>708</td>
<td>950</td>
</tr>
</tbody>
</table>

**Figure B.6:** Plots for Obj, data in table B.4

**Figure B.7:** Plots for data in table B.4
Data Analysis

The optimized results in table B.2 (answer report) shows that the optimal positioning of the Lift Rod to the Lower Link, LLLR, is as close to the hitch points as possible, $L_2=975\text{mm}$. This results in an optimal force distribution on the linkage-system, but to the cost of a greater length of the Upper Arm and by so, increasing stresses due to bending. By extending the length of the Upper Arm for the required Movement Range, the design increases in weight and by so becomes less cost effective. Therefore, a compromise is desired as an optimal final result. Graphs in figure B.6 shows that the force target function, Obj, increases gradually for the scenarios 1-4, where it increases more rapidly for the scenarios 5-7. Considering the side load caused by the angle of the lift rods ($\beta$), the attachment point is constrained by the alignment in a side view plane. With the angle $\beta$ set as a constraint not to exceed $5^\circ$, the length results in a range of 600-800mm for the distance $L_6$. Figure B.7 shows that the force $R$, acting on the upper attachment, suffer minimized loading in between loading scenario 3 and 4, where $L_2$ equals 675-775mm. For optimizing the total mass of the system, key is decreasing of the length of the upper arm. From a stress point of view, minimizing the length of the upper arm increases the tension in the lift rod but not to the same extent as it reduces the stresses in the upper arm due to bending. It is concluded to position the Lift Rod in the Lower range of scenarios 3 to 4. This to optimize the mass reduction while constraining to stay within the linear section of plot Obj in figure B.6. Initial values are set to: $L_2=700\text{mm}$ and $L_6=630\text{mm}$.

In conclusion the Lift rod is set to be positioned on the boundary of the constraints for the moving pattern. The distance from LLP to LLLR is set to 700mm and the distance from UAP to UALR is set to 630mm.
APPENDIX C
C.1 FEA

Autodesk inventor FEA is used to perform simple FEA analysis. Appendix C only gives a brief documentation of the load scenarios that has been concluded decisive critical scenarios. All analysis is carried out with numerous loading scenarios and boundary constraints.

C.2 FEA Approximations

Further note that a boundary constraint for fixed pin joints is not available using Inventor FEA version 9.0. This limits the accuracy of the resulting values where as the material stresses at the pivoting points can only be considered as a rough estimate. Each component with a pin joint is evaluated for a variety of boundary constraints to give sufficient data for an approximating value. The combination of evaluating either hole fixed or applied bearing load is mainly used. Forces applied as a baring load is estimated to give a more realistic result, where the fault lays in that the deflection is not restricted to a pin scenario. For a fixed scenario the boundary surface of the hole obtain a resulting moment.

C.3 Hydraulic Cylinder

C.3.1 Hydraulic Cylinder - Bottom Attachment

Load scenario
The boundary constraint is set for the top plane, connecting to the hydraulic cylinder, to be fixed where the load is exerted as a bearing load at the pin joint. Note that the boundary constrains is not ideal with the top plane approximated as fixed and the hole suffering a bearing load.

Max Load Scenario - 212kN

C.3.2 Hydraulic Cylinder - Upper Attachment

Load Scenario
The boundary constraints are set for the bottom plane, connecting to the hydraulic cylinder rod, to be fixed where the load is exerted as a bearing load at the pin joint. Also the surrounding weld is considered as a fixed plane. Note that the boundary constrains is not ideal with the bottom plane approximated as fixed and the hole suffering a bearing load. Also note that Upper Attachment is actually a welded assembly but is here analysed as one component with a homogenous material.

Max Load Scenario - 212kN
### C.4 Upper Arm Load Scenario

Maximum load on the upper arm is exerted at a fully raised positioning. This scenario is however unlikely to experience spikes unlike when an implement is in use. This scenario with the upper arm level with the ground is therefore set a decisive.

#### Max Load Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Forces calculated in [kN]</th>
<th>Converted to FEA coordinate system</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fully raised:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{UALR} = \begin{bmatrix} F_{2, xz} \cos (\beta) \cdot \cos (\alpha) \ -F_{2, yz} \cos (\beta) \cdot \sin (\alpha) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 47 \ -152 \end{bmatrix}$</td>
<td>$F_{UALR, rot} = M_{rot, UA} \cdot F_{UALR} = \begin{bmatrix} -74 \ -141 \end{bmatrix}$</td>
</tr>
<tr>
<td>$F_{UAHC} = \begin{bmatrix} \frac{F_3}{2} \cos (\phi_3) \ \frac{F_3}{2} \cdot \sin (\phi_3) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 193 \end{bmatrix}$</td>
<td>$F_{UAHC, rot} = M_{rot, UA} \cdot F_{UAHC} = \begin{bmatrix} 136 \ 136 \end{bmatrix}$</td>
</tr>
<tr>
<td>$F_{UAPL} = \begin{bmatrix} -R_{XL} \ -R_{ZL} \end{bmatrix}$</td>
<td>$\begin{bmatrix} -40 \ -34 \end{bmatrix}$</td>
<td>$F_{UAPL, rot} = M_{rot, UA} \cdot F_{UAPL} = \begin{bmatrix} -52 \ 4 \end{bmatrix}$</td>
</tr>
<tr>
<td>$F_{UAPR} = \begin{bmatrix} -R_{XR} \ -R_{ZR} \end{bmatrix}$</td>
<td>$\begin{bmatrix} -7 \ -199 \end{bmatrix}$</td>
<td>$F_{UAPR, rot} = M_{rot, UA} \cdot F_{UAPR} = \begin{bmatrix} -145 \ -136 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

*For detailed calculations, please refer to appendix A*

**Rotation matrix fully raised:**

$$M_{rot, UA} = \begin{bmatrix} \cos (\chi_{UA}) & -\sin (\chi_{UA}) \\ \sin (\chi_{UA}) & \cos (\chi_{UA}) \end{bmatrix}$$

$$\chi_{UA} = -\phi_4 = -45^\circ$$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Forces calculated in [kN]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level to the ground:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{UALR} = \begin{bmatrix} F_{2, xz} \cos (\beta) \cdot \cos (\alpha) \ -F_{2, yz} \cos (\beta) \cdot \sin (\alpha) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 36 \ -125 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$F_{UAHC} = \begin{bmatrix} \frac{F_3}{2} \cdot \cos (\phi_3) \ \frac{F_3}{2} \cdot \sin (\phi_3) \end{bmatrix}$</td>
<td>$\begin{bmatrix} 16 \ 121 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$F_{UAPL} = \begin{bmatrix} -R_{XL} \ -R_{ZL} \end{bmatrix}$</td>
<td>$\begin{bmatrix} -28 \ 4 \end{bmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$F_{UAPR} = \begin{bmatrix} -R_{XR} \ -R_{ZR} \end{bmatrix}$</td>
<td>$\begin{bmatrix} -40 \ -121 \end{bmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>
Fully lowered: Forces calculated in [kN]

Forces calculated in [kN] Converted to FEA coordinate system

\[
F_{UALR} = \begin{bmatrix} F_{2,yz} \cos(\beta) \cdot \cos(\alpha) \\ -F_{2,yz} \cos(\beta) \cdot \sin(\alpha) \end{bmatrix} = \begin{bmatrix} 22 \\ -107 \end{bmatrix} \quad F_{UALR,rot} = M_{rot,UA} \cdot F_{UALR} = \begin{bmatrix} 57 \\ -93 \end{bmatrix}
\]

\[
F_{UAHC} = \begin{bmatrix} \frac{F_3}{2} \cdot \cos(\varphi_3) \\ \frac{F_3}{2} \cdot \sin(\varphi_3) \end{bmatrix} = \begin{bmatrix} 11 \\ 91 \end{bmatrix} \quad F_{UAHC,rot} = M_{rot,UA} \cdot F_{UAHC} = \begin{bmatrix} -21 \\ 90 \end{bmatrix}
\]

\[
F_{UAPL} = \begin{bmatrix} -R_{xL} \\ -R_{xL} \end{bmatrix} = \begin{bmatrix} -16 \\ 13 \end{bmatrix} \quad F_{UAPL,rot} = M_{rot,UA} \cdot F_{UAPL} = \begin{bmatrix} -19 \\ 7 \end{bmatrix}
\]

\[
F_{UAPR} = \begin{bmatrix} -R_{xR} \\ -R_{xR} \end{bmatrix} = \begin{bmatrix} -29 \\ -89 \end{bmatrix} \quad F_{UAPR,rot} = M_{rot,UA} \cdot F_{UAPR} = \begin{bmatrix} 3 \\ -94 \end{bmatrix}
\]

Rotation matrix fully lowered:

\[
M_{rot,UA} = \begin{bmatrix} \cos(\chi_{LA}) & -\sin(\chi_{LA}) \\ \sin(\chi_{LA}) & \cos(\chi_{LA}) \end{bmatrix}
\]

\[
\chi_{LA} = -\varphi_3 = -20^\circ
\]

For detailed calculations, please refer to appendix A

C.5 Lower Link

Load Scenario

Maximum load on the Lower Links occurs at a fully lowered positioning. The resulting forces are calculated for this scenario and converted to the FEA coordinate system.

Max Load Scenarios

Fully raised: Forces calculated in [kN] Converted to FEA coordinate system

\[
LL_p = \begin{bmatrix} L_x \\ -L_y \end{bmatrix} = \begin{bmatrix} 108 \\ 38 \end{bmatrix} \quad LL_p,rot = M_{rot,TL} \cdot LL_p = \begin{bmatrix} 107 \\ -41 \end{bmatrix}
\]

\[
LL_{HP} = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} = \begin{bmatrix} -61 \\ -191 \end{bmatrix} \quad LL_{HP,rot} = M_{rot,TL} \cdot LL_{HP} = \begin{bmatrix} -170 \\ -105 \end{bmatrix}
\]

\[
F_{LLL} = \begin{bmatrix} F_{2,yz} \cos(\beta) \cdot \cos(\alpha) \\ F_{2,yz} \cos(\beta) \cdot \sin(\alpha) \end{bmatrix} = \begin{bmatrix} -47 \\ 152 \end{bmatrix} \quad F_{LLL,rot} = M_{rot,TL} \cdot F_{LLL} = \begin{bmatrix} 64 \\ 146 \end{bmatrix}
\]
For detailed calculations, please refer to appendix A

Rotation matrix:

\[
M_{\text{rot, LL}} = \begin{bmatrix}
\cos(\chi_{LL}) & -\sin(\chi_{LL}) \\
\sin(\chi_{LL}) & \cos(\chi_{LL})
\end{bmatrix}
\]

\[
\chi_{LL} = \phi_j = 40^\circ
\]

### Level to the ground: Forces calculated in [kN]

\[
LL_P = \begin{bmatrix}
L_x \\
-L_y
\end{bmatrix} = \begin{bmatrix}
115 \\
-36
\end{bmatrix}
\]

\[
LL_{HP} = \begin{bmatrix}
-F_x \\
-F_y
\end{bmatrix} = \begin{bmatrix}
-79 \\
-89
\end{bmatrix}
\]

\[
F_{LLL} = \begin{bmatrix}
-F_{2,xyz} \cdot \cos(\beta) \cdot \cos(\alpha) \\
F_{2,xyz} \cdot \cos(\beta) \cdot \sin(\alpha)
\end{bmatrix} = \begin{bmatrix}
-36 \\
125
\end{bmatrix}
\]

For detailed calculations, please refer to appendix A

<table>
<thead>
<tr>
<th>Fully raised: Forces calculated in [kN]</th>
<th>Converted to FEA coordinate system</th>
</tr>
</thead>
</table>
| \[
LL_P = \begin{bmatrix}
L_x \\
-L_y
\end{bmatrix} = \begin{bmatrix}
105 \\
-85
\end{bmatrix}
\] | \[
\overset{\text{Rotated}}{LL_{P,rot}} = M_{\text{rot, LL}} \cdot LL_P = \begin{bmatrix}
133 \\
-23
\end{bmatrix}
\] |
| \[
LL_{HP} = \begin{bmatrix}
-F_x \\
-F_y
\end{bmatrix} = \begin{bmatrix}
-83 \\
-21
\end{bmatrix}
\] | \[
\overset{\text{Rotated}}{LL_{HP,rot}} = M_{\text{rot, LL}} \cdot LL_{HP} = \begin{bmatrix}
-62 \\
-59
\end{bmatrix}
\] |
| \[
F_{LLL} = \begin{bmatrix}
-F_{2,xyz} \cdot \cos(\beta) \cdot \cos(\alpha) \\
F_{2,xyz} \cdot \cos(\beta) \cdot \sin(\alpha)
\end{bmatrix} = \begin{bmatrix}
-22 \\
107
\end{bmatrix}
\] | \[
\overset{\text{Rotated}}{F_{LLL,rot}} = M_{\text{rot, LL}} \cdot F_{LLL} = \begin{bmatrix}
-71 \\
82
\end{bmatrix}
\] |

For detailed calculations, please refer to appendix A

Rotation matrix:

\[
M_{\text{rot, LL}} = \begin{bmatrix}
\cos(\chi_{LL}) & -\sin(\chi_{LL}) \\
\sin(\chi_{LL}) & \cos(\chi_{LL})
\end{bmatrix}
\]

\[
\chi_{LL} = \phi_j = -29^\circ
\]