Project work report

MATERIAL MODELS FOR THE DEFORMATION AND FAILURE OF HIGH STRENGTH STEEL

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August 2008 to January 2009
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LIU-IEI-TEK-G--09/00095--SE
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Abstract

The aim of this work is to study the behaviour of high strength steel sheets considering anisotropy and non-proportional loading.

This report gives a review of some yield criteria and failure models. Some methods used to get parameters from tensile tests are also given.

Finally the first two steps of the study are exposed at the end of the report.
Acknowledgment

The work presented here is a project work performed at the Division of Solid Mechanics at Linköping University.

The work is a contribution to research work done by Rikard Larsson and Oscar Björklund, concerning the properties and influence of non-proportional loading on high strength steel sheets.

A special thanks should be given to my supervisor Prof. Larsgunnar Nilsson and my co-workers Rikard Larsson and Oscar Björklund for all their help and guidance during the project.

I would also like to thank all the Ph.D. students and other diploma workers at the division for all their support.

My stay at Linköping University would not have been such a pleasure without the good atmosphere at the Division of Solid Mechanics (and the Fikas on Fridays !). Thanks for fruitful discussions and friendship.

Finally, I would like to thank Thomas and my family for supporting me during the years.

Linköping in January 2009

Katia Boni
Chapter 1

Introduction

1.1 Background

The design of automotive structures is today to a large extent driven by simulations of its behaviour in its intended usage, and even of the forming process of its behaviour. The quality of these simulations to a large extent relies on the quality of the material models and their associated material data. Thus one needs to know the characteristics of the materials as precisely as possible: behaviour under a certain load, Young modulus, R-value, etc. This is also important in order to avoid making defective parts during a forming process.

In order to determine these values, one needs to perform various material tests: tests in shear stress, for plane stress, tensile tests with material prestrained in one direction with respect to the rolling direction, etc.

1.2 Aim of the project

The aim of this project is to characterize different high-strength steels such that their behaviour can be described by the material models. The project also aims at analysing the influence of non-proportional straining. All parameters of the materials will be obtained by results from tensile tests. These tests will be done in different directions with respect to the rolling direction (0° and 90° to the rolling direction)

The data obtained will especially be used in simulations of forming processes and crash tests.

1.3 Materials studied

Three materials have been studied:

- DOCOL 1200 M
- DOCOL 600 DP
- HyTens 1000
The first two materials are provided by the SSAB company. DOCOL are advanced high strength steels that make passengers cars and trucks lighter and safer.

DOCOL M grades are fully martensitic cold reduced steels. They are characterized by good formability at high strength levels combined with good weldability. The number 1200 corresponds to the minimum tensile strength level (expressed in $N/mm^2$).

DOCOL DP grades are high strength dual-phase steels, with a 2-phases structure that gives a combination of very high strength, far beyond conventional mild steel, whilst retaining formability.

The third material is provided by the OUTOKUMPU company. HyTens is the trademark for a new product family from OUTOKUMPU. HyTens is a high strength steel which is characterized by its maintenance-free surface and a good weldability. Whatever strength level, HyTens grades retain good formability (better than both carbon steel and aluminium). HyTens 1000 is a martensitic stainless steel, where the number 1000 indicates the strength level.

More information about these materials can be found on the companies’ websites.
Part I

Theory
Chapter 2

Material characterizations

In this chapter, some fundamental relations will be given.

2.1 Elasticity and plasticity

Experience shows that all solid materials deform when they are subjected to an external load. It was also shown that up to a certain limiting load, called the elastic limit, the material recovers its original dimensions when the load is removed. This phenomena is the so called elastic behaviour of the material.

On the contrary, when the elastic limit is exceeded, the material undergoes a permanent deformation when the load is removed. We say that the material undergoes plastic deformation.

In the following some basic assumptions and formulas will be given for the both cases, elastic and plastic behaviour.

2.1.1 Elastic behaviour

For most materials, the deformation is proportional to the load as long as the load does not exceed the elastic limit. This relationship is known as Hooke’s law. In general, this relation is stated as stress is proportional to strain. This can be written as:

\[ s = E e \]  \hspace{1cm} (2.1)

where \( s \) is the average stress or engineering stress, \( e \) is the average linear strain or engineering strain and the constant \( E \) is the modulus of elasticity or Young’s modulus.

This relation is illustrated by Figure 2.1 where just the elastic part of the flow curve (strain-stress curve) is represented.

However, if we want to relate the stress tensor \( \sigma \) to the strain tensor \( \varepsilon \), Equation (2.1) will change. Indeed, if a tensile force is then applied in the \( x \) direction, it will give a deformation along that axis but it will also involve some deformation in the tranversal \( y \) and \( z \) directions. Thus, stress-strain relations were developed for a three dimensionnal state of stress. The equations (2.2) are constitutive equations and are called Hooke’s generalized law.

\[ \sigma = C : \varepsilon \]  \hspace{1cm} (2.2)

where the components of \( \sigma \) and \( \varepsilon \) are
In particular, if the material is linear elastic and isotropic, we get

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2 \mu \varepsilon_{ij} \quad (2.5)$$

where $\lambda$ and $\mu$ are the Lamé's constants and $\delta_{ij}$ is the Kronecker's delta.

The Lamé's coefficients can be expressed as

$$\lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \quad (2.6)$$

and

$$\mu = \frac{E}{2(1 + \nu)} \quad (2.7)$$

where $E$ is the Young's modulus and $\nu$ is the Poisson's ratio.

More informations about elasticity are given by [1]. More details about tensors notation and their properties are developed by [2].

### 2.1.2 Plastic behaviour

When the stress reaches the elastic limit, the material starts to flow plastically and then an irreversible deformation occurs. Thus, Hooke's law is not valid anymore and a strain-stress relation is more difficult to find. However yield criteria and constitutive equations can be found for simple cases. Some of them will be presented in this part.
In the following, it is considered that the strain is the sum of an elastic and a plastic part (cf. (2.8)):

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]  \hspace{0.5cm} (2.8)

where \( \varepsilon^e \) is the elastic part, which can be determined by the relations (2.2) and where \( \varepsilon^p \) is the plastic part, which is equal to zero when the stress state is elastic.

**Uniaxial case**

In the case of elastic, ideally plastic material (cf. Figure 2.1.2 (a)), \( \sigma^Y \) does not change during the plastic deformation. So, it is easy to define the yield criterion as well as the strain. The initial value of the yield strength \( \sigma_0^Y \) is chosen as equal to the elastic limit \( \sigma^Y \) or the ultimate stress \( \sigma_u \), which is the maximum stress attained in hardening test.

However, most materials start to strain-harden (i.e. the stress to produce continued plastic deformation increases with increasing the strain) when the current elastic limit is reached. Therefore, the yield locus changes during the plastic deformation: it can be moved (kinematic hardening: cf. Figure 2.4) or it can grow (isotropic hardening: cf. Figure 2.3). Thus, \( \sigma^Y \) becomes a function of \( \varepsilon^p \). To approximate this function, we can consider a linear hardening (cf. Figure 2.1.2 (b)).

![Figure 2.2: Idealized flow curves: Hardening approximations: (a) Ideal plastic, (b) Linear hardening](image1)

![Figure 2.3: Isotropic hardening of the von Mises criterion](image2)
One way to approximate the hardening is to use the *power law*:

\[
\sigma^y = K(\varepsilon_0 + \varepsilon)^n \tag{2.9}
\]

where \(\varepsilon_0^y\) is the strain at initial yielding, \(K\) is a material constant and \(n\), the strain-hardening coefficient, is the slope of a log-log plot of equation (2.9). Note that \(K\) and \(n\) are material parameters.

The strain hardening coefficient can be calculated by:

\[
n = \frac{(\varepsilon_0 + \varepsilon)}{\sigma} \frac{d\sigma}{d\varepsilon} \tag{2.10}
\]

Another approximation is the *Ludwik curve* which is described by

\[
\sigma^y = \sigma_0^y \left( \frac{E\varepsilon}{\sigma_0^y} \right)^n \tag{2.11}
\]

where the parameters are defined as before.

Yet another approximation is the *Ramberg-Osgood equation* which expresses \(\varepsilon\) as

\[
\varepsilon = \frac{\sigma}{E} \left[ 1 + \alpha \left( \frac{\sigma}{\sigma_0^y} \right)^{n-1} \right]; \quad n < 1 \tag{2.12}
\]

where \(\alpha = K \left( \frac{\sigma_0^y}{E} \right)^{n-1}\), \(K\) are \(n\) are material parameters defined as above.

There are also more complex hardening rules which are not detailed here. More approximations of hardening can be found in Lemaitre and Chaboche [3] and Chakrabarky [4].

**Multiaxial case**

As we have seen so far, the yield stress of uniaxial plasticity defines the elastic domain in uniaxial stress case (cf. Figure 2.5). The generalization of this concept to the multiaxial case is the *yield criterion*.

Well known criteria are the von Mises and the Tresca criteria.
The von Mises criterion is based on an energetical study. It proposes that yielding would occur when the von Mises equivalent stress,

$$\sigma^{VM} = \sqrt{\frac{3}{2} S_{ij} S_{ij}}$$  \hspace{1cm} (2.13)

where $S_{ij}$ is the deviatoric part of the stress, i.e. $S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij}$ reaches a critical value $\sigma^Y$.

In the space of the principal stresses, this criterion can be written as:

$$\sigma^{VM} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \leq \sigma^Y_0$$ \hspace{1cm} (2.14)

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses.

Represented in the space of the principal stresses, this limit surface is a cylinder whose axis is the hydrostatic axis $\sigma_1 = \sigma_2 = \sigma_3$. The representation of the von Mises criterion in the principal stresses plane is given on Figure 2.5.

According to Tresca, the plasticity is not due to plastic work but due to the shear stress. That is why it is also called the Maximum Shear stress criterion. It is expressed as follows:

$$\sigma^T = \text{max}(|\sigma_1 - \sigma_2|, |\sigma_3 - \sigma_2|, |\sigma_1 - \sigma_3|)$$ \hspace{1cm} (2.15)

where $\sigma^T$ is the equivalent Tresca stress. The Tresca criterion (in the principal stresses plane) is represented on Figure 2.6.
Both criteria are for ductile materials. We can also note that the Tresca criterion is more conservative than the von Mises one.

Another yield criterion is the the Barlat yield criterion Barlat YLD2000 [10], which can be formulated as:

\[ \Phi' + \Phi'' = 2\sigma^m \]

where

\[ \Phi' = |s_1 - s_2|^m \]

and

\[ \Phi'' = |2s_2 + s_1|^m + |2s_1 + s_2|^m \]

where \( s_1 \) and \( s_2 \) are the principal values of the stress deviator. The exponent \( m \) is a material parameter (it is associated with the material crystal structure). The Figure 2.7 illustrates this criterion.

![Figure 2.7: The Barlat YLD2000](image)

**Necking**

Necking, also called *plastic instability*, generally begins at the maximum load during the tensile deformation of a ductile metal, where the increase in stress becomes greater than the increase in the load-carrying ability of the metal due to strain hardening. This condition of instability is defined by:

\[ \frac{d\sigma}{d\varepsilon} = \sigma \]

However, a different type of necking is found for a tensile specimen with rectangular cross-section that is cut from a sheet. For the latter, when the width is much greater than the thickness there are two types of tensile flow instabilities (cf. Figure 2.8):
• **Diffuse necking**
  It occurs according to the relation above. It may end by a fracture but is often followed by a second instability phenomenon:

• **Localized necking**
  The neck is a narrow band with a width about equal to the sheet thickness inclined at an angle to the specimen axis, across the width of the specimen.

![Figure 2.8: Diffuse and localized necking in a tensile sheet specimen](image)

### 2.2 Isotropy and anisotropy

All the relations we have seen so far are valid for a special case. Indeed, until now we have assumed that the material is **isotropic**, which means that the properties of the material are the same in all directions. However, in many materials, properties are **anisotropic**. Thus, they vary with orientation and directions with respect to the initial system of axes.

So, in this section the influence of anisotropy in the relations given previously will be studied.

#### 2.2.1 Elastic anisotropy

In the generalized case Hooke’s law may be written as

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \]  

where \( C_{ijkl} \) is the elastic stiffness. \( C_{ijkl} \) are fourth-rank tensor quantities.

If we expanded Equation (2.20), we would get nine equations, each with nine terms, which leads to 81 constants in all. But thanks to symmetry properties of the strain and stress tensors, only 21 independent elastic constants are needed.

It can be deduced from these equations that, comparing to the isotropic case, both normal strains and shear strains are able to contribute to a normal stress.

The components of the \( C \) tensor have to be determined experimentally. More details about elastic stiffness tensor can be found in [5].

#### 2.2.2 Plastic anisotropy

There are many anisotropic yielding criteria. But only two of them will be developed in the following.
Hill [9] formulated the von Mises yield criterion \((2.14)\) for an anisotropic material. It corresponds to orthotropic anisotropy in which three planes of symmetry are conserved during the hardening of the material. This criterion can be written as:

\[
F (\sigma_y - \sigma_z)^2 + G (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \quad (2.21)
\]

where \(x, y, z\) are the directions of anisotropy and \(F, G, H, L, M\) and \(N\) are constants defining the degree of anisotropy. It has been shown that these constants can be linked to yield stresses in the \(x, y\) and \(z\) directions, \(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}\) through the equation:

\[
F = \frac{1}{2} \left( \frac{1}{(\sigma_y)^2} + \frac{1}{(\sigma_z)^2} - \frac{1}{(\sigma_x)^2} \right) \quad L = \frac{1}{2} (\tau_{yz}^2)
\]

\[
G = \frac{1}{2} \left( \frac{1}{(\sigma_z)^2} + \frac{1}{(\sigma_x)^2} - \frac{1}{(\sigma_y)^2} \right) \quad M = \frac{1}{2} (\tau_{xz}^2)
\]

\[
H = \frac{1}{2} \left( \frac{1}{(\sigma_x)^2} + \frac{1}{(\sigma_y)^2} - \frac{1}{(\sigma_z)^2} \right) \quad N = \frac{1}{2} (\tau_{xy}^2)
\]

In the principal axes system this criterion becomes:

\[
F (\sigma_2 - \sigma_3)^2 + G (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 = 1 \quad (2.23)
\]

Hill has been working on his criterion for years. The other versions of the Hill criterion and other criteria for the anisotropic case can be found in [6] and [7].

A representation of the Hill criterion and the von Mises criterion is given in Figure 2.9.
2.2.3 R-values

Anisotropy is often described by the *Lankford parameter*, which measures the *normal anisotropy* [5]. It is defined as

\[
R = \frac{\varepsilon_w}{\varepsilon_t} = \frac{\ln(w_0/w)}{\ln(t_0/t)} \quad (2.24)
\]

where \( w_0 \) and \( w \) are respectively the initial and the final widths and \( h_0 \) and \( h \) are the initial and the final thicknesses. Thus \( \varepsilon_w \) and \( \varepsilon_t \) are the strains in the width and thickness directions, respectively.

\( R \) is also often noted:

\[
R = \frac{d\varepsilon_w}{d\varepsilon_t} \quad (2.25)
\]

For some materials, the \( R \) value may change during the deformation. Since most rolled sheets show a variation of elastic and plastic properties with orientation in the plane of the sheet, it is usual to replace the \( R \)-value by the average \( R \)-value, denoted \( \bar{R} \), which is defined as

\[
\bar{R} = \frac{R_0 + 2R_{45} + R_{90}}{4} \quad (2.26)
\]

where the indices indicate the orientation, in degrees, with respect to the rolling direction of the sheet.

Some yield criteria require these values. For example, the Hill criterion seen previously now becomes:

\[
\sigma_x^2 - \frac{2R_0}{1+R_0} \sigma_y \sigma_x + \frac{R_0 + R_{90}}{R_{90}(1+R_0)} \sigma_y^2 + \frac{(2R_{45} + 1)(R_0 + R_{90})}{R_{90}(1+R_0)} \tau_{xy}^2 = \sigma_0^2 \quad (2.27)
\]

Another criterion which also needs \( R \)-values is the anisotropic version of Barlat YLD2000 [8]. Indeed, the anisotropic case of the Barlat criterion seen in Section 2.1.2 is then handled by augmenting the occurrence of \( s \) (the stress deviator) in \( \Phi' \) and \( \Phi'' \) by the linear transformations \( C' \) and \( C'' \). In fact, the Equation (2.16) is kept but this time:

\[
\Phi'' = \left| X'_1 - X'_2 \right|^m \quad (2.28)
\]

and

\[
\Phi'' = \left| 2X''_2 + X'_1 \right|^m + \left| 2X''_1 + X'_2 \right|^m \quad (2.29)
\]

where in matrix form

\[
\{ X' \} = [C'] \{ s \} \{ L' \} \{ \sigma \} \quad (2.30)
\]

\[
\{ X'' \} = [C''] \{ s \} = [L''] \{ \sigma \} \quad (2.31)
\]

The matrices \( L' \) and \( L'' \) can be expressed in terms of the anisotropy coefficient \( \alpha_i \) (reduced to 1 in the isotropic case), according to

\[
\begin{bmatrix}
L'_{11} \\
L'_{12} \\
L'_{21} \\
L'_{22} \\
L'_{33}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
2 & 0 & 0 \\
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 2 & 0 \\
0 & 0 & 3
\end{bmatrix} \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_7
\end{bmatrix} \quad (2.32)
\]
and

$$\begin{bmatrix}
L_{11}'' \\
L_{12}'' \\
L_{21}'' \\
L_{22}'' \\
L_{33}''
\end{bmatrix} = \frac{1}{9} \begin{bmatrix}
-2 & 2 & 8 & -2 & 0 \\
1 & -4 & -4 & 4 & 0 \\
4 & -4 & -4 & 1 & 0 \\
-2 & 8 & 2 & -2 & 0 \\
0 & 0 & 0 & 0 & 9
\end{bmatrix} \begin{bmatrix}
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
\alpha_6 \\
\alpha_8
\end{bmatrix}$$

(2.33)

To determine the anisotropy coefficients $\alpha_i$, eight material tests need to be done. Input data could be the yield stresses $\sigma_{90}^Y$, $\sigma_{45}^Y$ and $\sigma_{90}^Y$ and the Lankford coefficients $R_{00}$, $R_{45}$ and $R_{90}$. More details about this criterion has been developed by Mikael Jansson [8].

A way to describe planar anisotropy is the Barlat-Lian criterion [13], which yield function is described as

$$a|K_1 + K_2|^M + a|K_1 - K_2|^M + C|2K_2|^M = 2\bar{\sigma}^M$$

(2.34)

$$K_1 = \frac{\sigma_{xx} + h\sigma_{yy}}{2}$$

(2.35)

$$K_2 = \sqrt{\left(\frac{\sigma_{xx} - h\sigma_{yy}}{2}\right)^2 + p^2\sigma_{xy}^2}$$

(2.36)

where $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$ are the plane components in orthotropic axes, $\bar{\sigma}$ is the effective stress (uniaxial yield stress in the rolling direction) and $a$, $c$, $h$ and $p$ are material constants.

If $M$ is assumed to be known, only three parameters are independent. Two simple methods can be used to determine the value of $a$, $c$, $h$ and $p$.

In the first method, yield stresses calculated by the Bishop and Hill model [14] for different loading conditions are used. For instance, if $\sigma_{90}$, $\tau_{S2}$ and $\tau_{S1}$ are the yield stresses for uniaxial tension in the transverse direction, shear such that $\sigma_{yy} = -\sigma_{xx} = \tau_{S2}$, $\sigma_{xy} = 0$ and shear such that $\sigma_{xx} = \sigma_{yy} = 0$, $\sigma_{xy} = \tau_{S1}$, then:

$$a = 2 - c = \frac{2 \left( \frac{\bar{\sigma}}{\tau_{S2}} \right)^M - 2 \left( 1 + \frac{\bar{\sigma}}{\sigma_{90}} \right)^M}{1 + \left( \frac{\bar{\sigma}}{\sigma_{90}} \right)^M - \left( 1 + \frac{\bar{\sigma}}{\sigma_{90}} \right)^M}$$

$$h = \frac{\bar{\sigma}}{\sigma_{90}}$$

$$p = \frac{\bar{\sigma}}{\tau_{S1}} \left( \frac{2}{2a + 2M^c} \right)^{1/M}$$

(2.37)

Figure 2.10 presents the tricomponent plane stress yield surface from Barlat-Lian criterion calculated with the Bishop and Hill model. On this figure $S = \sigma_{xy}/\bar{\sigma}$ represents the normalized shear stress.

The second method for determining $a$, $c$, $h$ and $p$ is to use $R$ values obtained from uniaxial tension tests in three different directions, for instance $R_{00}$, $R_{45}$ and $R_{90}$. This method has the advantage of taking $R$ values which are widely used parameters and which are obtained from a simple test on real materials or readily calculated from polycristalline models. Thus $a$, $c$ and $h$ can be written
\[ a = 2 - c = 2 - 2 \sqrt{\frac{R_0 - R_{90}}{1 + R_0 + R_{90}}} \]
\[ h = \sqrt{\frac{R_0 - 1 + R_{90}}{1 + R_0 + R_{90}}} \]

\( p \) can not be calculated analytically. However, when \( a, c \) and \( h \) are known, a relationship between \( R \) value for uniaxial tension in a direction making an angle \( \Phi \) with the rolling direction and \( p \) does exist.

### 2.3 Failure

Failure cannot be foreseen as easily, because the inside state of the material is not known. However some criteria have been established in order to anticipate the emergence of failure. These criteria are expressed in terms of mechanical variables, such as strain, stress... and represented failure model. In all criteria, the failure model is a function valid until a certain limit above which failure is expected.

There are many failure criteria; in this report only a few of them will be presented.

#### 2.3.1 Stress dependent failure criteria

**Maximum principal stress criterion**

One of the most simple way to predict failure is to consider that failure occurs when the stress reaches a critical value. The Maximum principal stress criterion is based on this assumption. This criterion, also known as Coulomb, or Rankine criterion, is often used to predict the failure of brittle materials. It only depends on the principal stresses.
It states that failure occurs when the maximum principal stress reaches either the uniaxial tension strength \( \sigma_t \), or the uniaxial compression strength \( \sigma_c \). Thus failure is not expected as long as the inequality (2.39) holds,

\[
-\sigma_c \leq \max(\sigma_1, \sigma_2, \sigma_3) \leq \sigma_t
\]

(2.39)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses.

In the principal stresses plane this criterion is represented by a cube, and as long as the principal stresses state stays inside the cube, failure is not expected. Figure 2.11 illustrates this criterion in the principal stress plane.

In the principal stresses plane this criterion is represented by a cube, and as long as the principal stresses state stays inside the cube, failure is not expected. Figure 2.11 illustrates this criterion in the principal stress plane.

**Figure 2.11: Maximum principal stress criterion**

**Mohr’s failure criterion**

The *Mohr’s failure criterion*, also known as the *Coulomb-Mohr criterion* or *internal-friction theory*, is based on the famous *Mohr’s Circle*. Mohr’s theory is often used in predicting the failure of brittle materials, and is applied in cases of 2D stress.

For this criteria too, only the two parameters \( \sigma_c \) and \( \sigma_t \) are required. But in this case, shear stress is also considered in order to predict failure.

The left circle is for uniaxial compression; it has the radius \( \frac{\sigma_c}{2} \) and its center is at the point \( \left( \frac{-\sigma_c}{2}, 0 \right) \). Likewise, the right circle is for uniaxial tension; its radius is \( \frac{\sigma_t}{2} \) and its center is located at the point \( \left( \frac{\sigma_t}{2}, 0 \right) \).

Mohr’s theory suggests that failure occurs when the Mohr’s Circle actual stress state at a point in the body exceeds the envelope created by the two Mohr’s circles for uniaxial tensile strength and uniaxial compression strength. This envelope is shown by two lines represented on Figure 2.12.

**Figure 2.12: Mohr’s failure criterion**
On Figure 2.12, the middle Mohr’s Circle represents the maximum allowable stress for an intermediate stress state. If the middle circle goes beyond the enveloppe, failure is expected. In the principal stresses plane, this criterion is represented as follows (Figure 2.13:

![Mohr’s failure criterion and maximum stress criterion in principal stresses plane](image)

Considering the figure 2.13 it can be concluded that the maximum stress criterion is less conservative than the Mohr’s failure one.

### 2.3.2 Strain dependant criteria

#### Maximum strain criterion

Just like the maximum stress criterion, there is also a maximum strain criterion. In this criterion it is established that failure is not expected as long as the maximum principal strain value is lower than a critical value $\varepsilon_f$ which is considered as a material parameter, i.e.:

$$\max(\varepsilon_1, \varepsilon_2, \varepsilon_3) \leq \varepsilon_f$$

(2.40)

where $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are the principal strains.

#### The forming limit diagram

The forming limit diagram (FLD) is often used in the analysis of forming processes in order to determine how close to failure the material is. This diagram was first developed by Keeler-Backhoften and Goodwin in the 60’s, c.f. Stoughton and Zhu [11].

The main part of this diagram is to determine the forming liming curve (FLC). The FLC is experimentally constructed and is represented on a graph which axes are the first and the second principal strains, see Figure 2.14.

One drawback of the FLD is that it is only valid when the loading path is proportionnal, which means that the ratio of the plastic strain must be constant throughout the forming process. Thus, if any pre-strain or non-linear loading is used, a new FLC is needed.
2.3.3 Shear instability

Plastic deformation is caused by slip on certain preferred slip systems. In order to make it possible to get plastic deformation, the shear stress needs to exceed a certain critical value $\tau_c$ which depends on the material.

Bressan and Williams [12] suggest a shear instability criterion that says that the plastic strain in the $X_t$-direction (Figure 2.15) should be equal to zero.

Due to the transformation of a second order tensor, the strain in the $X_t$-direction expressed in the main strain components becomes

$$d\varepsilon^p_{X_t} = \sin^2 \theta d\varepsilon^p_1 + \cos^2 \theta d\varepsilon^p_3 = 0 \quad (2.41)$$

This equation can be rewritten as

$$\cos 2\theta = \frac{d\varepsilon^p_1 + d\varepsilon^p_3}{d\varepsilon^p_1 d\varepsilon^p_3} \quad (2.42)$$

if the plastic volume is constant, i.e. $d\varepsilon^p_1 + d\varepsilon^p_2 + d\varepsilon^p_3 = 0$, and with $\beta = \frac{d\varepsilon^p_2}{d\varepsilon^p_1}$ this expression
becomes
\[ \cos 2\theta = -\frac{\beta}{2 + \beta} \]  \hfill (2.43)

If the same rotation as for the strain is done in the Mohr’s circle for the stresses (Figure 2.16), the following equation is obtained:

\[ \sin 2\theta = \frac{\tau_c}{\sigma_1} \]  \hfill (2.44)

Figure 2.16: Mohr’s circle

By using Equations (2.43) and (2.44) one finally obtains:

\[ \sigma_1 = \frac{2\tau_c}{\sqrt{1 - \left(\frac{\beta}{2 + \beta}\right)^2}} \]  \hfill (2.45)

where \( \sigma_1 \) is the largest principal stress, \( \tau_c \) is the critical shear stress, which is determined by experiments, and \( \beta \), as shown above, is a relationship between the strains in the plane.
Chapter 3

The tensile test of sheet metal

In this chapter, some assumptions about tensiles tests of sheet metal will be given.

A tensile test, also known as tension test, is probably the most fundamental type of mechanical test which can be performed on a material. Tensile tests are simple, relatively inexpensive, and fully standardized. By pulling on something, it can be determined very quickly how the material will react to forces being applied in tension. As the material is being pulled, its strength along with how much it will elongate can be found.

3.1 Test for E-modulus

For most tensile testing of materials, it can be noticed that in the initial portion of the test (cf. Figure 3.1), the relation between the applied force, or load, and the elongation the specimen exhibits, is linear. In this linear region, the line obeys the relation defined as Hooke’s Law where the ratio of stress to strain is a constant. $E$ is the slope of the line in this region where the engineering stress $s$ is proportional to the engineering strain $e$ and is called the Modulus of Elasticity or Young’s Modulus. This modulus is a measure of the stiffness of the material.

We can note that the greater the modulus, the smaller the strain resulting from the application of a given stress.

![Strain-Stress curve](image)

Figure 3.1: Strain-Stress curve
3.2 Tests for yield strength

A value called yield strength of a material is defined as the stress applied to the material at which plastic deformation starts to occur while the material is loaded.

For some materials (e.g., metals and plastics), the departure from the linear elastic region cannot be easily identified. Therefore, an offset method to determine the yield strength of the tested material is allowed. An offset is specified as a percentage of strain. For metals, the commonly used value is 0.2%. On Figure 3.2, the stress \( \sigma \) that is determined from the intersection point \( r \) when the line of the linear elastic region (with slope equal to Modulus of Elasticity) is drawn from the offset \( m \) becomes the Yield Strength by the offset method.

![Figure 3.2: The offset method](image)

3.3 Tests for R-Values

As defined in the previous chapter, the R-value \( R \) is the ratio between the total width \( \varepsilon_w \) and total thickness strain \( \varepsilon_t \):

\[
R = \frac{\varepsilon_w}{\varepsilon_t} \tag{3.1}
\]

In order to get this value, the width and the length of the sample are measured during the tensile test. Thus, the longitudinal and the width strain are obtained. Then, by assuming volume constancy (see Equation (3.2)), the thickness strain can be found. Thus the R-value can be calculated.

\[
\varepsilon_t = -(\varepsilon_w + \varepsilon_l) \tag{3.2}
\]

where \( \varepsilon_l \) is the length strain.

One other way to get the R-value is to calculate

\[
R = \frac{k_w}{k_t} \tag{3.3}
\]
Figure 3.3: One way to get the R-value

where $k_w$ and $k_t$ are respectively the slopes of the width and the thickness strain functions of the longitudinal strain (Figure 3.3).
Chapter 4

Influence of non-proportional straining

One main objective of this project is to determine the influence on plastic yielding and failure of non-proportional straining. Indeed, the FLD is established with a proportional loading. What will happen if it is not the case (Figure 4.1)? Will the FLD be the same? Will the fracture occur before or after the curve?

Figure 4.1: Forming Limit Diagram and unproportional loading

Through this project, one will try to answer these questions and so in different directions with respect to the rolling direction.

To simulate unproportional loading, the sample will first be stretched through a direction, then they will be unloaded before being stretched again through another direction.
Part II

Tests and results
Chapter 5

First series of tests

In this chapter, a description of the procedure and the results of the first series of tests will be given.¹

5.1 Process description

The first series of tests consists to prestrain some specimens (Figure 5.1). These specimens were cut out from a sheet of metal in the direction 0° and 90° with respect to the rolling direction. We made two types of prestrainig for each direction:

- Prestraining 90% to necking
- Prestraining 45% to necking

In order to foresee the result and to avoid fracture, especially for 90% to necking, a pre-study and a computer simulation were made by Rikard Larsson and Oscar Björklund.

For the test, the specimen is clamped at both side by screws (Figure 5.1). The torque applied to the middle screw is 40 Nm, and 79 Nm for the other screw.

The machine motion and the elongation in the middle of the sheet were measured. The latter was measured on both side of the specimen in order to be sure that the elongation is symmetric along the specimen. We also checked that the specimen did not slip during the test.

The width and the thickness were measured before and after the tensile test on different points of the sample. Also, by marking some points on the sample, the elongation repartition along the sample could have been calculated.

5.2 Results

First of all, this series of test was useful to make samples prestrained in one direction, which will be used for the following tests. They also allowed to check the validity of the pre-study and to have a first view of the parameters such as anisotropic parameters, Young’s modulus and yield strenghts.

¹For a reason of delay with the tensile tests, only the material DOCOL 1200 M will be treated in the following.
Figure 5.1: Test process
Chapter 6

Second series of tests

In this chapter, the second step of this project will be described.

6.1 Process

The second series of tests is constituted of some reference tests and prestrained test. The reference tests are tensile tests done with new material (not prestrained) respectively in the 00°-, 45°- and 90°-directions with respect to the rolling direction.

The prestrained tests are tensile tests performed with prestrained material. Some samples used for these tests were cut from the big samples, so they are prestrained either in the 00°- or the 90°-direction. These were strained in another direction. However, to get samples strained twice in the 00°-direction or twice in the 90°- or 45°-direction with respect to the rolling direction, some new material was taken (no samples were cut in the direction they were prestrained). These samples were then loaded, unloaded and loaded again in the same direction.

During the tests, the elongation and the evolution of the width were measured.

6.2 Results

From the measures taken from the tensile tests, R-values were calculated.

Stress-strain curves were obtained (Figures 6.1 and 6.2)

From these curves the yield strengths were determined. To do that, the offset method was used with an offset equal to 0.2%.

For the new material the values obtained in the different direction are

<table>
<thead>
<tr>
<th>Direction (°)</th>
<th>Yield strength (MPa)</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>912</td>
<td>$\sigma_{\mathrm{Y}00}$</td>
</tr>
<tr>
<td>45</td>
<td>879</td>
<td>$\sigma_{\mathrm{Y}45}$</td>
</tr>
<tr>
<td>90</td>
<td>1017</td>
<td>$\sigma_{\mathrm{Y}90}$</td>
</tr>
</tbody>
</table>

For the samples prestrained at 1% (which corresponds here to 90% to necking) in the 00°-direction (the rolling direction), the yield strengths in the different direction are

<table>
<thead>
<tr>
<th>Direction (°)</th>
<th>Yield strength (MPa)</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>1222</td>
<td>$\sigma_{\mathrm{Y}00,00}$</td>
</tr>
<tr>
<td>45</td>
<td>977</td>
<td>$\sigma_{\mathrm{Y}00,45}$</td>
</tr>
<tr>
<td>90</td>
<td>661</td>
<td>$\sigma_{\mathrm{Y}00,90}$</td>
</tr>
</tbody>
</table>
Figure 6.1: Stress-strain curve for new material

Figure 6.2: Stress-strain curve for material prestained at 1% to the 00°-direction
Thanks to these results, it was possible to fit the Barlat-Lian criterion. This was done with Excel.

First, the values of $c$, $h$ and $p$ were fixed to 1. The value of $a$ was calculated through the formula (2.37) and $M$ is equal to 4. Then the value of $\bar{\sigma}$ (Equation (2.34)) is calculated through the Equations (2.35) and (2.36) for the yielding strength in each direction. Thus,

- for the 00°-direction, $\sigma_{xx} = \sigma_{yy}^0$, $\sigma_{xy} = 0$ and $\sigma_{yx} = 0$
- for the 90°-direction, $\sigma_{xx} = 0$, $\sigma_{xy} = 0$ and $\sigma_{yy} = \sigma_{90}^y$

For the 45°-direction the value of $\sigma_{xx}, \sigma_{xy}$ and $\sigma_{yy}$ have to be calculated because the value of $\sigma_{45}^x$ was taken from a plane turned from 45° with respect to the original one. Considering $\sigma_{x'x'}$ and $\sigma_{y'y'}$ the axes of this plane, it follows that $\sigma_{x'x'} = \sigma_{45}^x$ and that the value of $\sigma_{xx}, \sigma_{xy}$ and $\sigma_{yy}$ can be calculated by

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \sigma_{x'x'} & \sigma_{x'y'} \\ \sigma_{x'y'} & \sigma_{y'y'} \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \tag{6.1}$$

where in this case $\phi = 45°$.

Once the $\bar{\sigma}$ has been calculated for all the directions, the error $e_i = (\bar{\sigma}_i - \sigma_{ref})^2$, $i = 00, 45$, is calculated, where $\sigma_{ref}$ is usually equal to $\sigma_{00}^y$. Then $E$ is calculated, where $E$ is defined as

$$E = e_{00} + e_{45} + e_{90} \tag{6.2}$$

Normally, with the calculus made before, the error $e_i$ should be equal to zero. That’s why the optimized values of $a$, $c$, $h$, and $p$ can be found by minimizing $E$. Thus the Barlat-Lian criterion is fitted. The values which were found are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.04</td>
</tr>
<tr>
<td>$c$</td>
<td>0.96</td>
</tr>
<tr>
<td>$h$</td>
<td>0.90</td>
</tr>
<tr>
<td>$p$</td>
<td>1.07</td>
</tr>
</tbody>
</table>

In order to draw the yield surface, it is now considered that the stresses $\sigma_{xx}, \sigma_{xy}$ and $\sigma_{yy}$ can be expressed as

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = t \begin{pmatrix} \cos \theta \\ \sin \theta \\ 0 \end{pmatrix} \tag{6.3}$$

where $\theta$ is an angle going from $0°$ to $360°$ and where $t$ is chosen arbitrarily.

As previously, the error $E$ is calculated. By minimizing $E$, the values of $t$ for each angle are obtained. Thus the values of $\sigma_{xx}$ and $\sigma_{yy}$ for each angle are calculated, which allows to draw the yield surface (Figure 6.3).

By the same way, the parameters $a$, $c$, $h$ and $p$ and the yield surface for the prestrained samples were found:
Figure 6.3: Yield surface for new material

Figure 6.4: Yield surface for material prestrained in the rolling direction
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.01</td>
</tr>
<tr>
<td>c</td>
<td>1.00</td>
</tr>
<tr>
<td>h</td>
<td>0.98</td>
</tr>
<tr>
<td>p</td>
<td>0.95</td>
</tr>
</tbody>
</table>

To compare these results, the two curves were put on a same graph (Figure 6.5).

![Graph showing yield surfaces from new and prestrained material](image)

Figure 6.5: Yield surfaces from new and prestrained material

Thanks to this graph it can be deduced that the yield strength in the 00°-direction of the prestrained material is higher than the new material’s one.

However, the yield strength of the prestrained material is smaller in the 90°-direction. Besides, comparing the results from the tables, this affirmation is also true for the 45°-direction.

So a general hypothesis can be made: prestraining involves a bigger yield strength in the direction of the prestraining. Otherwise, the yield strength is smaller.

The validity of this hypothesis will be checked in other tests.
Chapter 7

Conclusion

Thanks to these first two series of tests, a first idea of the behaviour of the material under unproportional loading has been determined. Indeed, now it can be supposed that for a prestrained material, the yield strength is higher in the direction of the prestraining than the other direction. Of course some other tests will be made to check this assumption. Other tests will also give more accurate results about the material’s behaviour under unproportional loading.

Other tests, such as shear test or plane stress will improve the results concerning the material behaviour. Thus it will allow to have a very accurate modelling of DP1200.
Bibliography


